

A STUDY OF PUBLIC RENTING BICYCLE SYSTEM CONSIDERING LOCATION AND NUMBER OF BICYCLE

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ABSTRACT

This study investigates the planning of public bicycle rental system in Taichung City, Taiwan. We use Taichung's MRT stations as the candidates of bike rental station. By Combining the data from the Ministry of Transportation and other literatures, the demand of public bicycle can be estimated for each candidate station. The model tries to find the best locations to set up bicycle rental stations and the number of bicycle required for each station. This study will consider the following costs: bicycle acquisition cost, land cost of bicycle rental station, replacement cost, and bicycle rearrangement cost (*i.e.* a daily operation cost for arranging bicycle back to its initial location). The objective of this study is to minimize total cost by selecting suitable bicycle rental stations and arranging necessary bicycles for each station. In this study, we first develop a mathematical model and then construct a heuristic algorithm to solve the problem by using the simulated annealing logic. In addition, a sensitivity analysis is also conducted by considering different land costs and different distances required between stations.

Keywords: Public bike rental system, site selection, setup costs.

1. INTRODUCTION AND BACKGROUND

Due to the green environment considerations in recent years, a public bicycle rental system combined with public transportation systems is believed to be a better way to reduce air pollution and improve transportation efficiency. In designing a public bicycle rental system, several factors must be carefully evaluated such as user requirements, bike rental site selection and acquisition cost, bicycle requirement and arrangement for each rental location, system initial investment and operating costs.

In real world bicycle rental system, customer may rent a bike at one location and return it to another location. This situation, sooner or later, may cause some rental location having no space to return a bike or no bike to rent. Therefore, rearrangement of bicycle for each rental station back to its original location is necessary which may cause additional cost and should be taken into consideration.

This study tries to propose an effective bicycle rental system in the initial design stage. In this study, a mathematical model is developed to minimize overall cost, including bicycle acquisition cost, land cost for each rental station, replacement cost due to limited service stations installed, and bicycle rearrangement cost due to different rental and return point. A heuristic solution algorithm is then developed to solve a real world problem focusing on the public bicycle rental system of Taichung city in central Taiwan.

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2. LITERATURE REVIEW

History of public bicycle system evolves in three stages. The first stage was free public bicycle system which was developed in Amsterdam, Netherland in 1965. The second stage was coin rental public bicycle system developed in Copenhagen, Denmark in 1995. The third stage applies information technology to public bicycle rental system in Plymouth, England in 1996. In Taiwan, several cities have launched public bicycle rental system since 2009. Most of the systems accept credit card or membership card to activate the bicycle system in renting and returning operations.

2.1 Riding Distance of Public Bicycle System

European Commission (1999) indicates 2- to 8-min bicycle riding distance is an efficient equipment for public transportation. In addition, the average riding speed is 15 km h⁻¹ (Metropolitan Washington Council of Governments 2010). Therefore, the average riding distance is 0.5 to 2 km. This study will use 12 km, *i.e.* four times of 2 km, as the maximal distance between two bicycle rental stations.

2.2 Estimate Demand for Bicycle Rental Station

The study on Dublin, Ireland by Commins and Nolan (2011) indicates 3.5% population of 15 years old or older will use bicycle in their daily life. In Taichung city, Taiwan, 2.2% of 15 years old or older will use bicycle in their daily transportation. Lusk *et al.* (2014) indicates 30.9% of male bicycle users will rent public bicycle and the female users is 31.7%. Based on the above information, this study will use 0.6%, *i.e.* 2.2%* (30.9% + 31.7%)/2, population of 15 years old or older as the demand for each bicycle rental station.

2.3 Land Requirement and Candidate Location of Rental Station

Based on the data from the government of Taipei city, every 30 bicycles need 52 m², *i.e.* 26 m long and 2 m width, therefore, this study will use 2 m² per bicycle as the land requirement.

In the initial developing stage of Taipei city, most of the bicycle rental stations are located on subway stations which can be easily connected to other public transportation system. This concept will be applied in Taichung city which is the case study in this research.

2.4 Other Studies Related to Public Bicycle Rental system

Martinez *et al.* (2012) use mixed-integer programming to solve the public bicycle rental system in Lisbon, Portugal. Their study investigates locations of rental station and number of bicycle requirement. The objective of the model is to maximize profit through the revenues generated by the selected stations, setup cost of rental station, and bicycle acquisition cost. Frade and Ribeiro (2014) estimate demand of bicycle by considering population density, density of job opportunity, density of sale retailers, sightseeing attractions, and type of geology. Their study will develop an independent public bicycle system which combines with other transportation systems that can illustrated in a real city. Lin and Yang (2011) propose a mathematical model to minimize system cost by considering setup cost of rental station, construction cost of bike riding paths, and bicycle rearrangement cost under service level constraints.

3. MODEL CONSTRUCTION AND SOLUTION ALGORITHM

This study assumes that there are N candidate locations that can be selected as rental stations in a city. Each candidate location should serve local community with known population which will be used to estimate demand of bicycle. For the candidate location that is not selected as a rental station, the bicycle rental requirement will be replaced by other rental station which will generate a replacement cost. The replacement cost includes additional land cost and additional bicycle acquisition cost. For those locations selected as bicycle rental stations, distance between two stations must be longer than minimal distance requirement and shorter than maximal distance requirement.

Two types of bicycle return operations are considered in this study: (1) rent a bike from A station and return it to B station, and (2) rent and return a bike at the same station. These operations will change the number of bicycle availability at the end of each renting period. Therefore, bicycle rearrangement to its initial available number is necessary for each station. In this paper, the cost of rearrangement operation is called the rearrangement cost.

The objective of the model developed in this study is to minimize overall cost of public bicycle rental system. The overall cost includes: (1) setup cost of rental station including land cost and bicycle acquisition cost, (2) replacement cost for

the candidate location, and (3) cost of bicycle rearrangement.

3.1 Basic Assumptions and Limitations

This study is developed on the basis of following assumptions and limitations.

1. Candidate locations of public bicycle rental station are given and known. Rental station should be selected from candidate locations. The number of rental station to be selected is given and known. This number should be less than the number of candidate location.
2. Demand of bicycle is calculated by the population around the candidate location and percentage of user. Population of community and percentage of user are given and known.
3. If a candidate location is not selected, its bicycle rental requirement will be replaced by other rental station. This situation will initiate a replacement cost which consists of additional land cost and additional bicycle acquisition cost.
4. All unit costs are given and known. The unit cost includes bicycle acquisition cost per bicycle, unit land cost of each candidate location, replacement cost per bicycle, and rearrangement cost per bicycle per period (day).
5. Distance between any two candidate locations is given and known.
6. For each bicycle rental station, the renter can return bicycle to any station. Probability of returning destination is given and known. These probabilities can be used to calculate rearrangement cost.
7. At the end of each operation period (*i.e.* day), each station should arrange bicycle to its initial number. These operations will generate the rearrangement cost.
8. Each rental station has limitation on space which is represented by number of bicycle can be stored. Each bicycle requires 2 m² for parking.
9. For any rental station, distance to nearby station should be within 0.5 to 12 kilometer.
10. The public bicycle rental system should satisfy all rental requirements from all candidate locations.

3.2 Notations and Variables

In this subsection, several notations and variables are prepared and defined for the following mathematical model.

Notations and variables:

N	total candidate locations selected as public bicycle rental station.
i, j, k	candidate location number which is also the bicycle demand number. $i, j, k = 1, 2, \dots, N$
D_i	demand of bicycle for candidate location i . $i = 1, 2, \dots, N$
d_{ij}	distance from location i to location j . $i, j = 1, 2, \dots, N$, $i \neq j$ ($d_{ii} = 0$)
K	bicycle acquisition cost.
T_i	land cost per square meter for candidate location i . $i = 1, 2, \dots, N$
P	replacement cost per bicycle which includes bicycle acquisition cost.
CL_i	number of bicycle installed in candidate location i . $i = 1, 2, \dots, N$

- F_i the shortest distance from rental station i to other rental stations. $F_i = \min \{d_{ij}, j = 1, 2, \dots, N, i \neq j\}$
 SN number of rental station to be installed. SN is given and known. $SN < N$
 MD_k after replacement consideration, final demand of bicycle in rental station k . $k = 1, 2, \dots, N$
 R_{ik} if candidate location i is selected as rental station, R_{ik} is bicycle return percentage from rental station k to rental station i . $i, k = 1, 2, \dots, N$
 W_{jk} if candidate location j is not selected as rental station, W_{jk} is bicycle return percentage from rental station k to candidate location j . $j, k = 1, 2, \dots, N, j \neq k$
 RC rearrangement cost per bicycle per period. (in this study one period is one operation day)
 DS periods in system life. DS is used to calculate overall rearrangement cost, *i.e.* change one period rearrangement cost to life time rearrangement cost.
 UG land requirement per bicycle (m^2)
 M a large positive number

Decision variables:

- X_i if candidate location i is selected as a rental station, then $X_i = 1$, otherwise $X_i = 0$. $i = 1, 2, \dots, N$
 S_{ij} if the candidate location j is not selected as a rental station and the bicycle demand of candidate location j is replaced by rental station i , then $S_{ij} = 1$, otherwise $S_{ij} = 0$. $i, j = 0, 1, 2, \dots, N, i \neq j$
 Y_i if the candidate location i is selected as a rental station and the bicycle of rental station i needs to be rearranged, then $Y_i = 1$, otherwise $Y_i = 0$. $i = 1, 2, \dots, N$

3.3 Model Construction

The objective function and constraints are developed as follows.

$$\text{Min. } Z = Z1 + Z2 \quad (1)$$

$$Z1 = \sum_{i=1}^N X_i [D_i K + UG \times D_i T_i] + \sum_{i=1}^N \sum_{j=1}^N S_{ij} [D_j P + UG \times D_j T_i] \quad (2)$$

$$Z2 = RC \times DS \times \sum_{i=1}^N Y_i \left\{ \sum_{i=1}^N X_i \left[\sum_{i=1}^N \sum_{j=1}^N \sum_{k=1}^N (R_{ik} MD_k + S_{ij} W_{ij} MD_k) - (X_i MD_i) \right] \right\} \quad (3)$$

$$\text{st. } \sum_{i=1}^N X_i = SN \quad (4)$$

$$\sum_{i=1}^N \sum_{j=1}^N S_{ij} = N - SN, i \neq j \quad (5)$$

$$\sum_{i=1}^N \sum_{j=1}^N S_{ij} X_j = 0, i \neq j \quad (6)$$

$$\sum_{i=1}^N \sum_{j=1}^N X_i S_{ij} = N - SN, i \neq j \quad (7)$$

$$\sum_{i=1}^N X_i D_i + \sum_{i=1}^N \sum_{j=1}^N S_{ij} D_i = \sum_{i=1}^N D_i \quad (8)$$

$$M(1 - X_i) + X_i F_i \geq 0.5, \quad \text{for all } i \quad (9)$$

$$X_i F_i \leq 12, \quad \text{for all } i \quad (10)$$

$$X_i D_i + \sum_{j=1}^N X_i S_{ij} D_j \leq CL_i, \quad i = 1, 2, \dots, N, i \neq j \quad (11)$$

$$X_i D_i + \sum_{j=1}^N S_{ij} D_j = X_i MD_i, \quad i, j = 1, 2, \dots, N \quad (12)$$

$$X_i \in \{0, 1\}, \quad i \in N \quad (13)$$

$$S_{ij} \in \{0, 1\}, \quad i, j \in N, i \neq j \quad (14)$$

$$Y_i \in \{0, 1\}, \quad i \in N \quad (15)$$

The objective function (Eq. (1)) is to minimize cost of public bicycle rental system which includes two parts as indicate in Eqs. (2) and (3). Equation (2) defines the land cost and bicycle acquisition cost for selected rental station after replacement and Eq. (3) for rearrangement cost by calculating bicycle number to be arranged at the end of each period.

The first constraint (Eq. (4)) states that the required rental stations should be selected. Equation (5) ensures there are $N-SN$ candidate locations to be replaced by rental stations. Equation (6) makes sure a selected location, *i.e.* a rental station, cannot be replaced. Equation (7) ensures total unselected location should be $N-SN$. Equation (8) confirms that all demands of candidate location should be satisfied. Equations (9) and (10) make sure that distance between rental stations is within limitation. Equation (11) ensures number of bicycle does not exceed a predefined maximal number in each rental station. Equation (12) confirms bicycle demand of any rental station coming from the location itself and the replaced location. Equations (13) to (15) ensure that all decision variables are 1 or 0 integers.

3.4 Solution Algorithm

An initial solution is generated randomly and satisfied all constraints as indicated in subsection 3.3. Since the real case study (Taichung city) presented in this study has 40 candidate locations and 35 rental stations to be selected from those candidates, the solution space will reach 658,008, *i.e.* $C(40, 35)$. In addition, if the possibility of replacement (or substitution) for five unselected locations is considered, the solution space will up to $658,000 \times (35)^5$. It becomes a NP hard problem in this case. Therefore, the solution algorithm is developed on the basis of simulated annealing (SA) logic. In each iteration of the improvement procedure, one of the selected rental stations will be

All cost data are summarized as follows. The bicycle acquisition cost is NT\$ 10,000 per bicycle and the replacement cost is NT\$ 15,000 per bicycle. The land cost is collected from websites provided by city government. Table 3 summarizes land cost per square meter for each candidate location. The rearrangement cost is NT\$ 500 per bicycle per day. Rearranging bicycle is a daily operation, however, the cost of objective function is count on the basis of life time. Therefore, this study should assume a reasonable life time, which is 10 years (3650 days). The rearrangement

cost will be changed from daily basis to life time basis. Distance data between any two locations is collected from the Google map using walking mode.

4.2 Solution and Discussion

Based on the data collected in subsection 4.1 and the solution procedure discussed in subsection 3.4, a computer program is then developed to solve the problem. Table 4 summaries the cost data of initial solution and final solutions.

Table 3 Land cost for each candidate location

Zone ID	Candidate Location ID	Land Cost (NT\$/m ²)	Zone ID	Candidate Location ID	Land Cost (NT\$/ m ²)	
6. West	BA1	112,000	8. North	O3	831,250	
	B1	162,750		G0	126,875	
	B2	195,125		G3	140,000	
	B3	169,750		G4	306,250	
	O1	271,250		G5	262,500	
	O2	87,000		G6	198,625	
	G8	214,375		T1	306,250	
	G8a	214,375		T2	297,500	
	G9	696,500		5. Central North	O4	168,875
	G10	530,250			O5	250,250
4. Central West	B5	514,500	O6		144,375	
	B6	315,875	O7	227,500		
	B7	133,000	3. Central South	O10	236,250	
1. Central	B8	183,750		O11	288,750	
	O8	262,500		T5	302,750	
	O9	336,000		T6	161,875	
	T4	336,000		G13	161,875	
2. Central East	B10	231,000	7. South	G10a	831,250	
	B11	121,624		G11	507,500	
	T3	107,625		G12	787,500	

Table 4 Cost comparison of initial solution and final solutions

Run No.	Total Cost (OFV)	Setup Cost (Land + Bicycle)	Replacement Cost (Land + Bicycle)	Rearrangement Cost	
Initial Solution	4,984,695,325	2,722,164,750	319,652,000	1,942,878,575	
2	Final Solution (Improvement %)	3,301,499,675* (33.76%)	1,955,821,250* (15.37%)	285,738,500** (0.68%)	1,059,939,925 (17.71%)
	Cost Improvement	1,683,195,650	766,343,500	33,913,500	882,938,650
6	Final Solution (Improvement %)	3,325,549,375 (33.27%)	2,061,267,250 (13.25%)	255,267,000 (1.29%)	1,009,015,125 (18.73%)
	Cost Improvement	1,659,145,950	660,897,500	64,385,000	933,863,450
5	Final Solution (Improvement %)	3,356,306,700 (32.66%)	2,129,205,250** (11.89%)	221,898,750 (1.96%)	1,005,202,700* (18.81%)
	Cost Improvement	1,628,388,625	592,959,500	97,753,250	937,675,875
4	Final Solution (Improvement %)	3,415,149,450 (31.48%)	2,061,267,250 (13.25%)	183,389,250* (2.73%)	1,170,492,950 (15.50%)
	Cost Improvement	1,569,545,875	660,897,500	136,262,750	772,385,625
3	Final Solution (Improvement %)	3,420,028,425 (31.38%)	2,061,267,250 (13.25%)	226,295,750 (1.87%)	1,132,465,425 (16.26%)
	Cost Improvement	1,564,666,900	660,897,500	93,356,250	810,413,150
1	Final Solution (Improvement %)	3,442,058,475** (30.94%)	2,061,267,250 (13.25%)	194,773,000 (2.51%)	1,186,018,225** (15.18%)
	Cost Improvement	1,542,636,850	660,897,500	124,879,000	756,860,350
Average Cost Improvement (Average Improvement %)	1,607,929,975 (32.25%)	667,098,833.3 (13.38%)	91,758,291.67 (1.84%)	849,022,850 (17.03%)	

Remark: 1. *The best one out of 6 runs, **The worst one out of 6 runs,
 2. Cost Improvement = Initial Cost – Final Cost,
 3. Improvement % = (Initial Cost – Final Cost) / Initial Cost × 100%

There are six independent runs based on different random number seeds and the best final solution is found in the run #2. In general, the solution procedure proposed in this study can improve overall cost up to 32% which comes from setup cost 13%, replacement cost 2%, and rearrangement cost 17%.

Form the best solution in run #2, Table 5 indicates cost in details for all 35 rental stations, which are selected from 40 candidate locations.

There are five rental stations replacing five unselected candidate locations, therefore, the replacement costs (land cost and bicycle acquisition cost) should be added in these rental stations. It is believed that if other requirements of replacing condition are considered, then the associated constraints should be included in

the mathematical model. In this study, the judgment only focuses on a “lower” replacement cost.

4.3 Sensitivity Analysis on Land Cost and Distance between Stations

For most large cities, the land cost is the major cost item in installation of a public bicycle rental system. This study conducts a sensitivity analysis assuming different land cost per square meter, *i.e.* 50%, 80%, 100% (the original case), 120%, and 150%. The results and cost data are presented in Table 6. It is found that the costs of rearrangement will change, however, the cost trend of rearrangement is different from the trend of land cost.

Table 5 The rental stations selected in run #2 (the best) and cost data

Station ID	Setup Cost	Replacement Cost (Station ID/Bicycle)	Rearrangement Cost	Sub Total
BA1	21,762,000	35,611,000 (G10a/149)	0	57,373,000
B1	31,201,500	0	40,648,225	71,849,725
B2	37,223,250	0	40,648,225	77,871,475
B3	32,503,500	0	40,648,225	73,151,725
O1	51,382,500	0	72,134,950	123,517,450
O2	17,205,000	0	72,134,950	89,339,950
G8	40,803,750	0	65,827,750	106,631,500
G8a	40,803,750	0	65,827,750	106,631,500
G10	99,556,500	0	65,827,750	165,384,250
B6	98,829,500	0	0	98,829,500
B7	42,504,000	0	0	42,504,000
B8	30,200,000	0	64,373,225	94,573,225
O8	42,800,000	0	95,859,950	138,659,950
O9	54,560,000	0	95,859,950	150,419,950
T4	54,560,000	105,798,000 (B5/154)	0	160,358,000
B10	48,144,000	0	24,223,225	72,367,225
B11	25,831,500	0	24,223,225	50,054,725
T3	22,975,500	36,149,250 (O3/157)	0	59,124,750
G0	41,408,750	0	0	41,408,750
G3	45,530,000	0	0	45,530,000
G4	97,732,500	0	0	97,732,500
G5	83,995,000	0	0	83,995,000
G6	63,938,250	0	0	63,938,250
T1	97,732,500	0	65,992,000	163,724,500
T2	94,985,000	0	65,992,000	160,977,000
O4	52,510,250	0	0	52,510,250
O5	77,085,500	0	0	77,085,500
O6	45,111,250	0	0	45,111,250
O7	70,215,000	0	0	70,215,000
O10	49,215,000	0	55,709,950	104,924,950
O11	59,925,000	0	55,709,950	115,634,950
T5	62,781,000	57,706,500 (G9/93)	0	120,487,500
T6	34,042,500	50,473,750 (G12/149)	0	84,516,250
G13	34,042,500	0	48,298,625	82,341,125
G11	152,725,000	0	0	15,2725,000
Total	1,955,821,250	285,738,500	1,059,939,925	3,301,499,675

Table 6 Sensitivity analysis on land cost per unit square meter

Scenario No.	Adjustment of Unit Land Cost (%)	Total Cost (OFV)	Setup Cost (Land+Bicycle)	Replacement Cost (Land+Bicycle)	Rearrangement Cost
(1)	50	2,323,070,263	1,166,349,875	134,403,750	1,022,316,638
(2)	80	3,039,207,775	1,657,427,800	208,613,400	1,173,166,575
Original	100	3,301,499,675	1,955,821,250	285,738,500	1,059,939,925
(3)	120	3,990,605,888	2,465,106,700	336,936,000	1,188,563,188
(4)	150	4,618,388,200	2,913,066,875	481,638,750	1,223,682,575

Table 7 Sensitivity analysis on distance between rental stations (see comments before)

Scenario No.	Distance Limitation (km)	Total Cost (OFV)	Setup Cost (Land + Bicycle)	Replacement Cost (Land + Bicycle)	Rearrangement Cost
Original	0.5 to 12	3,301,499,675	1,955,821,250	285,738,500	1,059,939,925
(1)	0.5 to 2	4,341,293,788	2,724,489,750	417,696,000	1,199,108,038

In design stage of public bicycle rental system, distance between rental stations is also an important issue; should be user friendly and convenient. Therefore, the following sensitivity analysis focuses on reducing distance from 0.5-12 km (the original case) to 0.5-2 km. The cost results are indicated on Table 7. Reducing distance between stations will increase overall cost up to 132% in this scenario. It is obviously that improving the convenience of renter will increase the installation cost. A more detailed investigation is deserved if a more user friendly system is considered.

5. CONCLUSIONS

This study suggests a mathematical model to select suitable rental stations for a public bicycle rental system from candidate locations. The objective function is to minimize costs including land cost and bicycle acquisition cost. The requirements of bicycle from the unselected locations are considered in the objective function as replacement costs. In addition, the daily operational cost of bicycle arrangement is also included in consideration. A heuristic solution algorithm is suggested to solve a real world NP hard problem.

A case study of Taichung city's public bicycle rental system is conducted to illustrate the model and solution heuristic. The solution results and sensitivity analysis are useful for system designer and management level. The analysis approach suggested in this study is worth of future researches.

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