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Numerical investigation for religious affiliation population dynamics model

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ABSTRACT

The curiosity of analyzing the impact of religious affiliation among different social communities in the modern era can derive significant contribution in terms of the role played by religion in human lives. To facilitate the investigation, a fully connected network represented by a set of ordinary differential equations (ODEs) reflects the religious affiliation population dynamics (RAPD) model is analyzed. The RAPD system comprising of three subpopulations: committed religious (CR), non-committed religious (NCR) and non-religious (NR) individuals. Transitions among these groups as well as spontaneous shifts are governed by probability based on social interactions. The efficacy of the Adams predictor-corrector method is exploited to generate the reference data set that portrays the real-world data dynamics. The reference data set is utilized as the benchmark for the actual data where case studies along with their related cases are formulated by varying the values of RAPD's parameters i.e., the probabilities of social pressures and spontaneous evolution. The comparative analysis is presented with the aid of employing various numerical techniques including backward differentiation formula (BDF), explicit and implicit Runge-Kutta (RK) to trace transitions and trends assessing absolute error (AE) in numerical results. For the purpose of evaluating the outcomes in terms of the accuracy, efficiency and robustness, solution plots illustrations along with the AE metric is presented. The efficiency in the performance of the various numerical techniques is reflected via AE values that shows that even the maximum error is negligible and below 10-7.

Keywords: Adams Numerical solver; Backward differentiation formula; Committed religious; Explicit Runge Kutta; Implicit Runge Kutta; Proximity Analysis; Religious affiliation population dynamics

NOMENCLATURE

AE:	Absolute error
BDF:	Backward differentiation formula
CR:	Committed religious individuals
NCR:	Non-committed religious individuals
NR	Non-religious individuals.
ODEs:	Ordinary differential equations
PS:	Place spirituality
RAPD:	Religious affiliation population dynamics
RK:	Runge-Kutta

1. INTRODUCTION

In Recent years, models of opinion convergence have increasingly focused on understanding belief formation through social networks and the interactions between like-minded individuals. On such approach, the religious affiliation population dynamics model integrates these two processes into a single framework with a parameter controlling the balance between social influence and affiliation. This model demonstrates a phase transition from diverse opinion to widespread as the parameter shifts [1], providing a powerful tool for analyzing social alignment patterns within communities on and spirituality continue to play a fundamental role in shaping values, behaviors and worldwide views for much of the global population [2-8].

The religious affiliation as a degree of freedom in human networks analyzing the growth of 18 religions from 1900 to 2000 has been studied that highlights competitive dynamics alongside the traditional "at birth" effect. The role of external influences is compared to crystal growth models, and opinion-based models with vector-like agents are suggested and investigated [9]. The phase transitions analysis in social opinion models under different noise levels, comparing two distinct approaches: one rooted in social impact theory and the other emphasizing agent's social influence. At high noise levels both models result in phases where no majority opinion emerges. In contrast , at low noise levels, differences between the two models become more pronounced. The second model driven by the influence of highly connected leaders exhibit unique behaviors and specific noise thresholds that mark phase transitions [10]. A scalable approach to opinion dynamics that examine the interaction at both individual and group level is also introduced. By incorporating group affiliations, this effectively captures key social patterns, including social consensus, polarization, and fragmentation. This framework connects individual

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actions to collect group behavior, offering a robust simulation of complex social dynamics [11]. Chaos theory and self-organization investigation has been applied, to address worldwide social, political, and environmental challenges. As these calls for the development of a social theory that combines insights from natural and social sciences indicating the importance of collaboration across discipline aiming for a suitable and resilient global future [12]. The process of radicalization within a population divided into core and sensitive subgroups which includes immigrant communities. Core agents hold rigid views while sensitive agents have the potential to adapt their views or become radicalized. Core agents can play a role in countering radicalization with their level of indolent influenced by the size of the sensitive subgroups and the actions of opposing groups [13]. The impact of homophily-driven evolution in social networks on the spread of opinion has been studied. Introducing a certain percentage of committed agents can accelerate consensus once a critical threshold is met causing consensus time to increase only logarithmically. The positive influence of committed agents applies across various interaction rules although these rules can still affect how consensus time scales [14].

A technique for detecting membership in covert groups through the use of classical and quantum holography has been explored. By applying this spectral analysis to these interactions , the unique signature of the group can be identified. This approach also enables the identification of unknown individuals by comparing their vocal spectra to those of establish group members [15]. Religion and spirituality play a central role in many lives deeply influencing values , behaviors and world views . while religious affiliation has declined in some western areas about 80% of the global population remains religious and this proportion is expected to increase further [16]. The issues and gaps highlighted in commentaries on place spirituality (PS), supporting the developmental pathways of this concept has been also explored and studied. The research on PS, examining how it connects with various religions such as Islam, Christianity, and Hinduism. Additionally, it considers whether PS should be understood as a transactional or transitional phenomenon. The research also defends both the compensation and correspondence attachment models, arguing for their relevance within the context of PS. These models are further integrated with the motivational systems framework to provide a more comprehensive understanding [17]. The psychology of religion examines whether a unique religious experience exists, linking spirituality to private transcendence and religion to institutions. Conversion and deconversion are experiential, influenced by cultural context. Mysticism, defined by self-loss and unity, is a key religious experience found in both traditional and spiritual settings [18]. Psychologist's interest in spirituality and religion requires a clear understanding of their characteristics. In this research, the propose criteria that acknowledge the similarities and differences, offering benchmarks for evaluating existing definitions [19], Psychology derived from the Latin psyche (soul), explores inner self and existential questions. Religion and spirituality are often sensitive topics that have gained attention in multicultural psychology. Recent research emphasizes their relevance in cultural diversity, covering definitions, historical views, and identity development. This chapter addresses ethical considerations and current trends related to these themes [20].

The relationship between religion and social change focusing on Islam and Muslims to better understand the role of social change within Islamic beliefs and its implications for social sciences has been studied in [21-25]. Furthermore, forecasting global social dynamics, across multiple domains and phase transitions that indicate critical shifts in societal organization and development also have been investigated in [26-30]. The nonlinear ordinary differential equations (ODEs) are solved using the efficient numerical techniques including Adams numerical solver [31-32], backward differentiation formula (BDF) [33-34], and Runge Kutta (RK) methods [35-37]. There has been a growing interest among researchers to employ numerical techniques that provides efficient, robust and accurate solutions for various social and economic models [38-39]. The ongoing research also implements Adams numerical approach to investigate the bi-directional nanofluids flow in the presence of Cattaneo–Christov double diffusion [40], childhood disease model [41], dengue disease [42], and Williamson fluid stretching flow model [43]. In this research exploiting the knacks of the various numerical techniques the model of the religious affiliation population dynamics (RAPD) system is numerically solved to present the efficient solutions along with the comparative analysis is also provided.

Salient features of the study on the religious affiliation population Dynamics (RAPD) model:

- Analyzes the impact of religious affiliation on social communities, emphasizing religion's role in modern society.
- Analysis based on fully connected network of ODEs to represent dynamics among three subpopulations: committed religious (CR), Non-committted religious (NCR), and non-religious (NR) individuals.
- Group transitions and spontaneous shifts are governed by probability based on social interactions, reflecting real world religious affiliation dynamics
- Adam's predictor-corrector method exploitation to generate reference data set that simulates real-world dynamics.
- Formulation of various case studies by adjusting and altering RAPD parameters, specifically probabilities of social pressure and spontaneous changes.
- Comparative analysis based on various methods including BDF, explicit and implicit RK to track transitions, trends and assess accuracy in terms of absolute error (AE) in numerical outcomes.
- Assessment of accuracy, efficiency and robustness of the numerical techniques employed for solving RAPD model via solution plots and AE metrics showing error is negligible.

The presented study is divided into the following sections: the first section provides an introduction to the concept and necessary literature based on the RAPD system. The remaining work is divided as follows: section 2 represents the mathematical formulation of the RAPD model, section 3 provides the results along with necessary interpretations, and section 4 represents the conclusion of the proposed strategy.

2. MATHEMATICAL FORMULATION

This section represents the mathematical modeling of the RAPD system using ODEs investigated in [44]. The synthetic data set is generated using Adams numerical method that simulates realistic religious population dynamics serving as the benchmark for the real-world data. In order to check the robustness of the employed numerical technique and for the sake of comparative analysis, the BDF, implicit and explicit RK methods are employed. The four case studies along with the associated three cases are formulated in such a way that the values of the RAPD model parameters are varied and adjusted accordingly that depicts the real-world dynamics in behavioral aspects. The four parameters are the probabilities of spontaneous transition and social pressures symbolized as: α, β, ρ, and γ whose values are altered to produce the diverse case studies of the RAPD model. Table 1 demonstrates the formulated diverse case studies and related cases of the RAPD model for better understanding and readability for the interested readers. For case study 1 the values of the ρ are varied and α is set to be 0.030, similarly case study 2 is formulated by varying the values of α, the value of α is set as 0.080 and the value of γ for case study 3 is varied, and finally for case study 4 the values of β is varied and α is set to be 0.150.

Mathematical model of religious affiliation population dynamics (RAPD) system is given in equation 1.

$$\begin{aligned} &CR'(t) = -\alpha CR(t) + \beta CR(t)NCR(t) + \rho CR(t)NR(t), \\ &NR'(t) = -\rho CR(t)NR(t) + \gamma NR(t)NCR(t), \\ &NCR'(t) = -\gamma NR(t)NCR(t) - \beta CR(t)NCR(t) + \alpha CR(t), \\ &\text{Initial conditions of the state variables:} \\ &CR(0) = 0.5, \quad NR(0) = 0.05, \quad NCR(0) = 0.45, \end{aligned} \tag{1}$$

Table 1 Parameter variation of RAPD model

Case study	Cases	Parameters				
		ρ (Social pressure of religious committed individuals)	γ (Probability of social pressure)	β (social pressure)	α (Spontaneous transitions)	t
S-1	1	0.05	0.10	0.12	0.030	500
	2	0.04	0.10	0.12	0.030	500
	3	0.06	0.10	0.12	0.030	500
S-2	1	0.05	0.10	0.12	0.049	5000
	2	0.05	0.10	0.12	0.050	5000
	3	0.05	0.10	0.12	0.051	5000
S-3	1	0.05	0.07	0.12	0.080	300
	2	0.05	0.08	0.12	0.080	300
	3	0.05	0.09	0.12	0.080	300
S-4	1	0.05	0.10	0.12	0.150	150
	2	0.05	0.10	0.14	0.150	150
	3	0.05	0.10	0.16	0.150	150

With the modifications and adjustments in the values of the parameters, and initial conditions of the three state variables i.e., *CR(t)*, *NR(t)*, and *NCR(t)*.as defined in the current section, the remaining RAPD system of equations can be similarly produced for each of the prescribed four case study along with the three associated cases. While few of the variants are demonstrated as follows:

Mathematical expressions of the RAPD system for the exploitation of the numerical schemes for case study 1 case 1 is given in equation 2 as:

$$\begin{aligned} &CR'(t) = -0.030CR(t) + 0.12CR(t)NCR(t) + 0.05CR(t)NR(t), \\ &NR'(t) = -0.05CR(t)NR(t) + 0.10R(t)NCR(t), \\ &NCR'(t) = -0.10NR(t)NCR(t) - 0.12CR(t)NCR(t) + 0.030CR(t), \\ &t \to 500 \end{aligned} \tag{2}$$

While for other few variants the mathematical representation is given in equations 3, 4, 5, 6, 7, 8, and 9 as follows:

Case study 1 case 3

$$\begin{aligned} &CR'(t) = -0.030CR(t) + 0.12CR(t)NCR(t) + 0.06CR(t)NR(t), \\ &NR'(t) = -0.06CR(t)NR(t) + 0.10R(t)NCR(t), \\ &NCR'(t) = -0.10NR(t)NCR(t) - 0.12CR(t)NCR(t) + 0.030CR(t), \\ &t \to 500 \end{aligned} \tag{3}$$

Case study 2 case 1

$$\begin{aligned} &CR'(t) = -0.049CR(t) + 0.12CR(t)NCR(t) + 0.05CR(t)NR(t), \\ &NR'(t) = -0.05CR(t)NR(t) + 0.10R(t)NCR(t), \\ &NCR'(t) = -0.10NR(t)NCR(t) - 0.12CR(t)NCR(t) + 0.049CR(t), \\ &t \to 3000 \end{aligned} \tag{4}$$

Case study 2 case 3

$$\begin{aligned} &CR'(t) = -0.051CR(t) + 0.12CR(t)NCR(t) + 0.05CR(t)NR(t), \\ &NR'(t) = -0.05CR(t)NR(t) + 0.10R(t)NCR(t), \\ &NCR'(t) = -0.10NR(t)NCR(t) - 0.12CR(t)NCR(t) + 0.051CR(t), \\ &t \to 3000 \end{aligned} \tag{5}$$

Case study 3 case 2

$$\begin{aligned} &CR'(t) = -0.080CR(t) + 0.12CR(t)NCR(t) + 0.05CR(t)NR(t), \\ &NR'(t) = -0.05CR(t)NR(t) + 0.08R(t)NCR(t), \\ &NCR'(t) = -0.08NR(t)NCR(t) - 0.12CR(t)NCR(t) + 0.080CR(t), \\ &t \to 300 \end{aligned} \tag{6}$$

Case study 3 case 3

$$\begin{aligned} &CR'(t) = -0.080CR(t) + 0.12CR(t)NCR(t) + 0.05CR(t)NR(t), \\ &NR'(t) = -0.05CR(t)NR(t) + 0.09R(t)NCR(t), \\ &NCR'(t) = -0.09NR(t)NCR(t) - 0.12CR(t)NCR(t) + 0.080CR(t), \\ &t \to 300 \end{aligned} \tag{7}$$

Case study 4 case 1

$$\begin{aligned}&CR'(t)=-0.049CR(t)+0.12CR(t)NCR(t)+0.05CR(t)NR(t),\\&NR'(t)=-0.05CR(t)NR(t)+0.10R(t)NCR(t),\\&NCR'(t)=-0.10NR(t)NCR(t)-0.12CR(t)NCR(t)+0.150CR(t),\\&t\to 150\end{aligned} \tag{8}$$

Case study 4 case 2

$$\begin{aligned}&CR'(t)=-0.049CR(t)+0.14CR(t)NCR(t)+0.05CR(t)NR(t),\\&NR'(t)=-0.05CR(t)NR(t)+0.10R(t)NCR(t),\\&NCR'(t)=-0.10NR(t)NCR(t)-0.14CR(t)NCR(t)+0.150CR(t),\\&t\to 150\end{aligned} \tag{9}$$

Case study 4 case 3

$$\begin{aligned}&CR'(t)=-0.150CR(t)+0.16CR(t)NCR(t)+0.05CR(t)NR(t),\\&NR'(t)=-0.05CR(t)NR(t)+0.10R(t)NCR(t),\\&NCR'(t)=-0.10NR(t)NCR(t)-0.16CR(t)NCR(t)+0.150CR(t),\\&t\to 150\end{aligned} \tag{10}$$

3. RESULT AND DISCUSSION OF THE PROPOSED METHODOLOGY

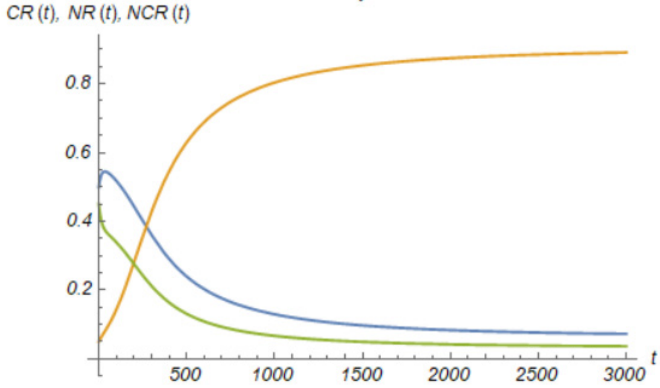
In this section, the results of the various numerical techniques employed to solve the mathematical model of religious affiliation are provided in Figure 3. Here in Figure 3, the first column represents the results for the Adam numerical method for different case studies, while the second column represents the BDF, Implicit and explicit Runge-Kutta (RK), for the second case study and its first case, case study 3: case 2, and case study 4: case3. These illustrations in Figure 3, parts a, b, c, d, e, and f, provide the comparison among the numerical techniques being employed to solve the RAPD model comprising of the three compartments, i.e., CR(t), NR(t), and NCR(t).

Figures 4, 5,6, 7, 8, and 9 are divided into two parts where part a (a:1, a:2, a:3) represents the solution plots for the three state variables *CR(t)*, *NR(t)*, and *NCR(t)* of the RAPD model solved with the numerical techniques that include the Adams numerical solver, BDF, implicit RK, and explicit RK. Part b of Figures 4 to 9 represents the absolute error (AE) analysis for the evaluation of the proposed strategy. The Adams numerical technique is used to generate the data that we used as the benchmark for the reference dataset because of the efficient results produced that depict the real-world religious affiliation dynamics from behavioral perspectives. For evaluation purposes, various numerical techniques have been employed to solve the RAPD model while comparing it with the reference dataset. The accuracy and efficiency of the acquired data sets are visualized via solution plots and evaluated using the AE metrics that show the gap between the reference data and each generated dataset with the employment of various numerical techniques used for the analysis.

The case studies are formulated by varying the values of the four parameters i.e., the probabilities of spontaneous transition and social pressures symbolized as: α, β, ρ, and γ, along with the related three cases for each case study presented. The parameter values are varied for three times to form the first case study i.e., case study 1-case 1, case study 1-case 2, case study 1-case 3 (variation in ρ values, time range *(t)* is taken as 0 to 500), on a similar pattern case study 2 with three cases are formulated with the variation in α values while t is set from 0 to 3000. For the third case study, the γ value is varied three times, and the t ranges from 0 to 300, giving case study 3 with case 1, case 2, and case 3. The value of β is varied for the fourth case study and *t*= 150. The explicit RK is employed to solve the RAPD model for its first case study and associated cases in Figure 4. Parts a:1 represent the solution plots for the CR (t), a:2 for NR(t), and a:3 for NCR(t), while parts b:1, b:2, and b:3 represent the AE analysis for the *CR(t)*, *NR(t)*, and *NCR(t)*. The reference dataset is compared with the efficient results produced by the explicit RK as the solutions overlap with each other, while this performance is evaluated with the aid of the AE metric. The range of the AE is minimum for the prescribed cases and each state variable. The overlapping solutions indicate the accurately produced results with the employment of the efficient numerical techniques, with the AE being minimum and below 10^{-6} , 10^{-7} for NR*(t)*, and 10^{-6} for *NCR(t)*. across all case studies.

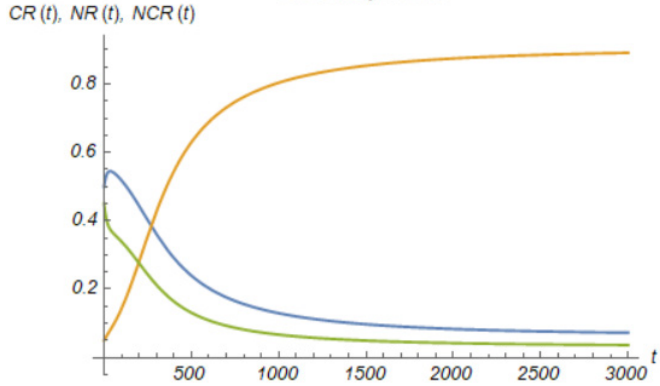
The visual representations of the solution dynamics of the religious affiliation population model solved with the exploitation of the BDF numerical technique are given in Figure 5. The efficiency of the proposed strategy is visualized with the evaluation of the error analysis for the *CR(t) NR(t)*, *NCR(t)*. that even the maximum error is minimum and ranges between 10^{-6} to 10^{-8} . This demonstrates the effectiveness of the BDF technique with a minimum margin of error in solving the RAPD model. The Figure 6 shows the solution plots of the RAPD model for third case study by employing implicit RK. The effectiveness of the accurate results produced by the implicit RK technique can be visualized via evaluation metrics in terms of AE analysis for the RAPD state variables. The comparative analysis between the actual and the estimated implicit RK solutions appears to overlap while this accuracy is quantified in terms of AE measure that ranges between 10^{-6} to 10^{-9} across all cases of the RAPD model's third case study. The solutions of the fourth case study of RAPD model are provided with the exploitation of the implicit RK in Figure 7, explicit RK in Figure 8, and BDF technique in Figure 9. All of the mentioned numerical techniques are compared with the results produced via Adams numerical technique that we referred to as the proxy of the actual dataset. The error is minimum and ranges between 10^{-6} to 10^{-11} for *CR(t)*, NR(t), and NCR(t) with the exploitation of implicit RK, while the error is also minimum with the employment of the explicit RK and BDF as AE value ranges from 10^{-4} to 10^{-9}, and 10^{-4} to 10^{-9} across all cases of the formulated fourth case study of the RAPD model. The obtained accuracy in the results reinforces the effectiveness of the employed numerical techniques in solving the complex, stiff, nonlinear systems while maintaining the minimum error threshold.

Adams numerical method



(a)

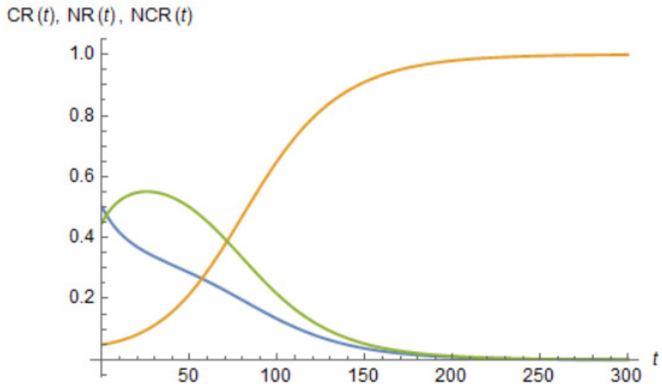
BDF



(b)

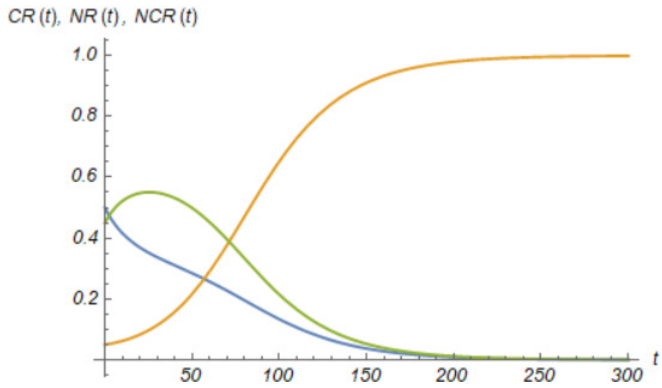
Case study 2 case 1

Adams numerical method



(c)

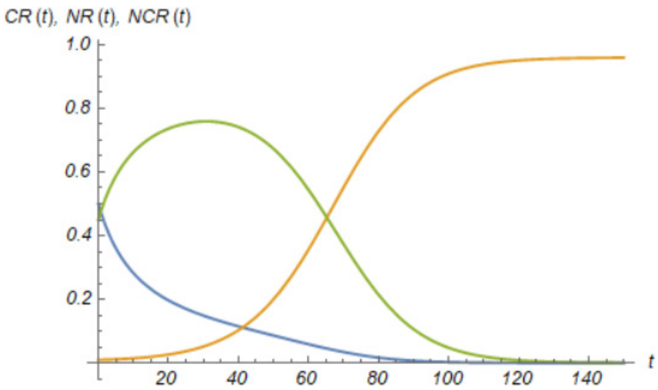
Implicit RK



(d)

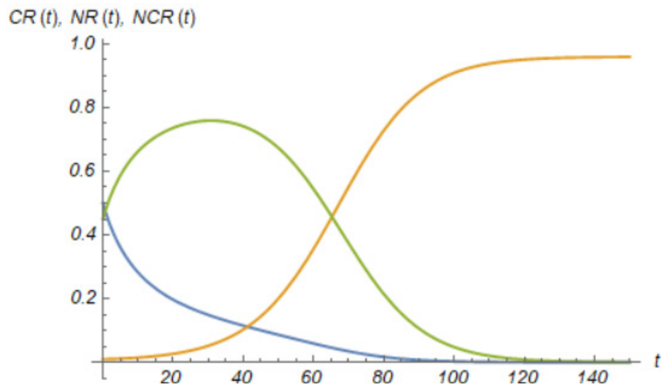
Case study 3 case 2

Adams numerical method



(e)

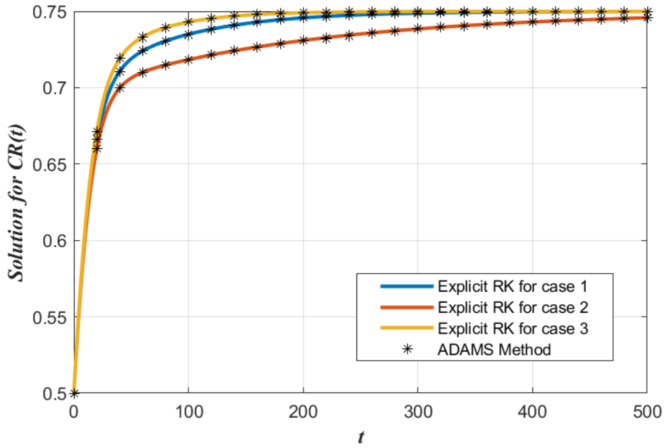
Explicit RK



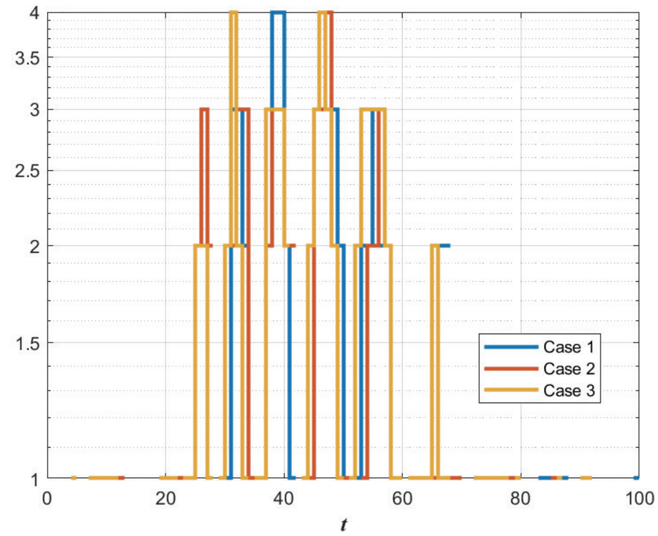
(f)

Case study 4 case 3

Fig. 3 Illustrations of the various numerical methods employed to solve the RAPD model.



(a:1)



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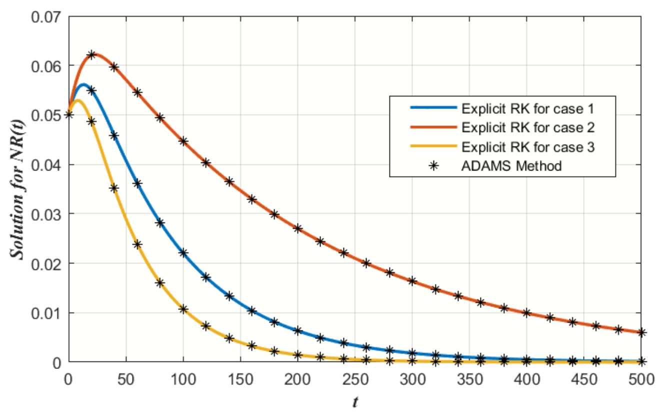
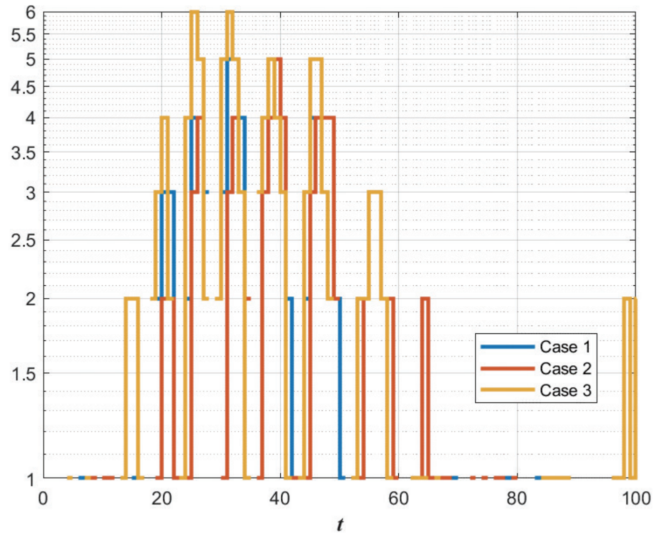


(a:2)



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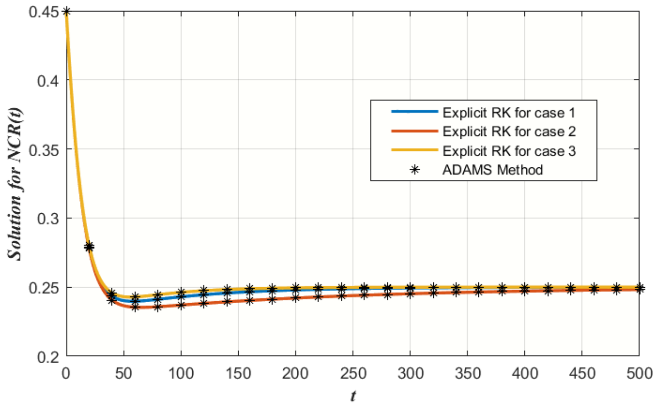

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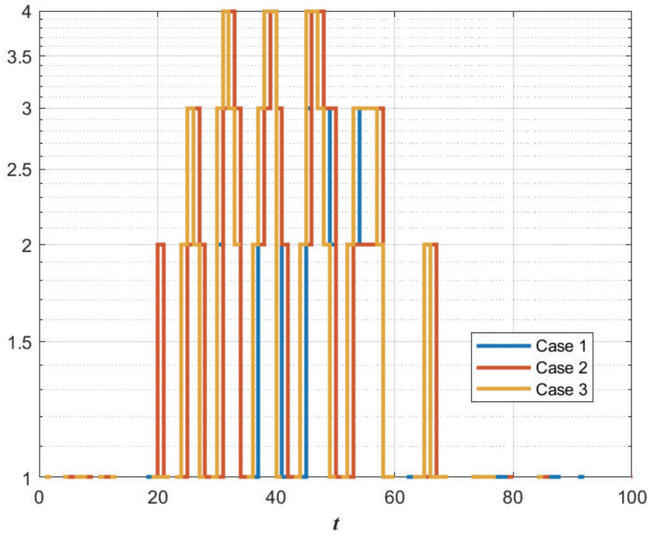

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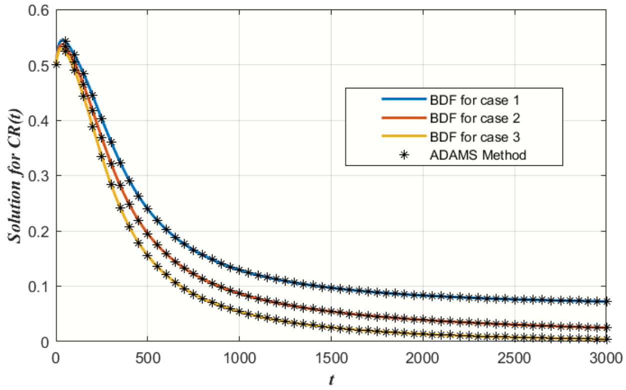
Fig. 4 Solution plots for RAPD model with the employment of Explicit RK numerical technique for case study 1 and associated cases.



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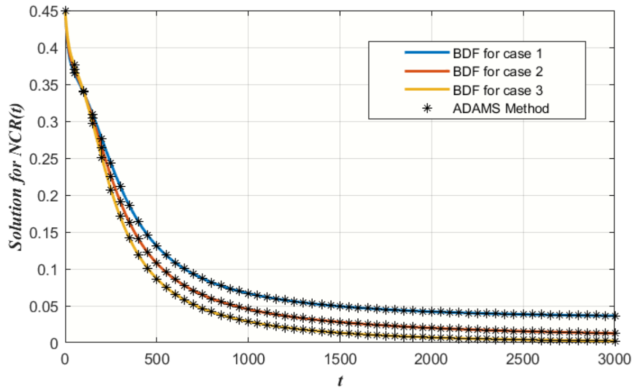

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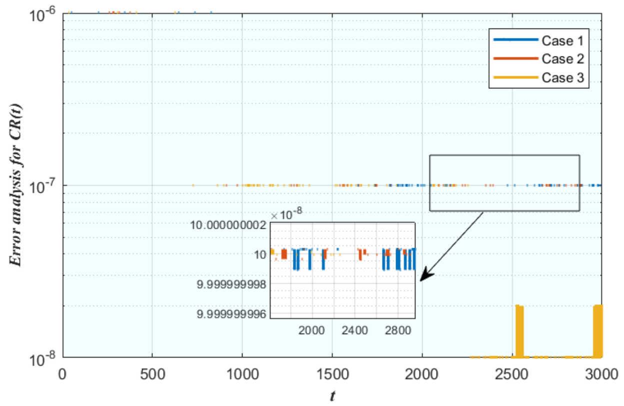

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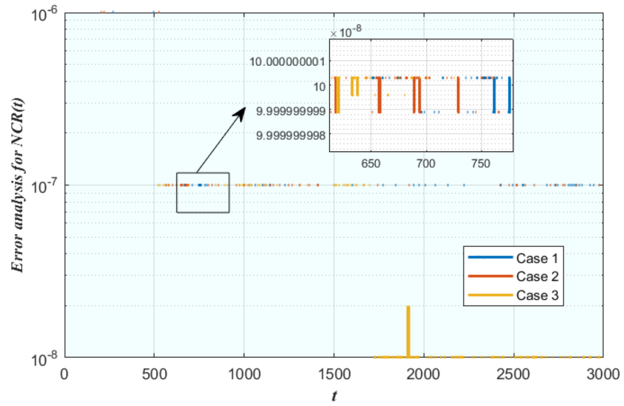

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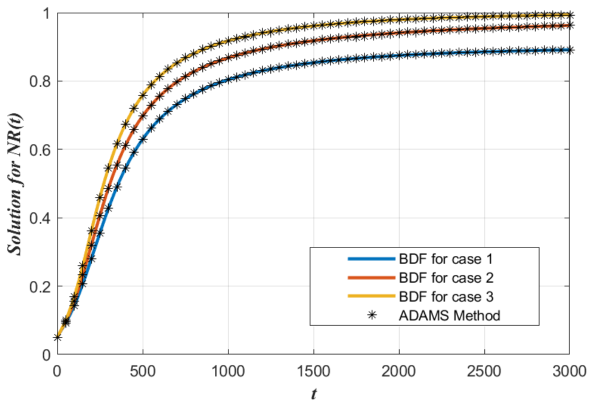

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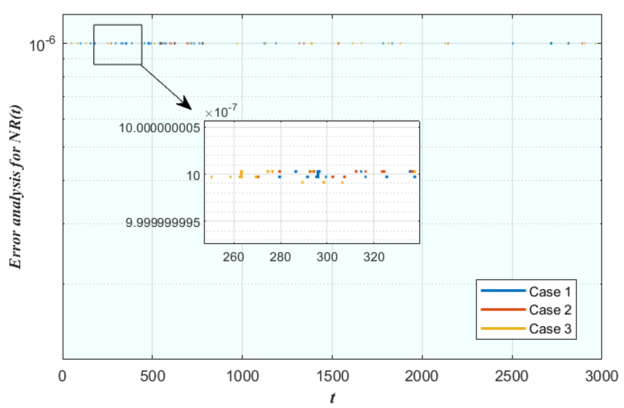
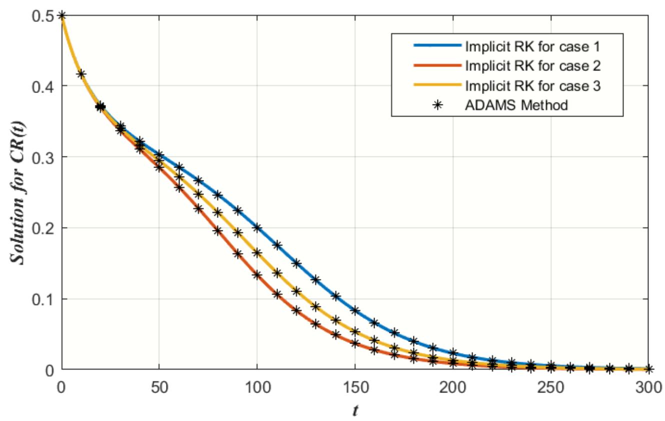
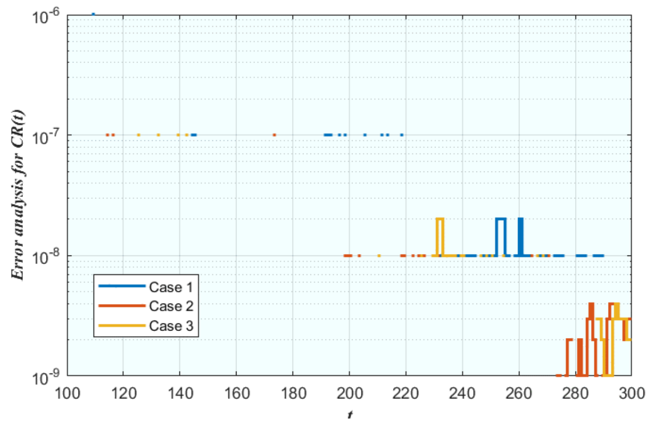


(b:2)

Fig. 5 Solution plots for RAPD model with the employment of BDF numerical technique for case study 2 and associated cases.



(a:1)



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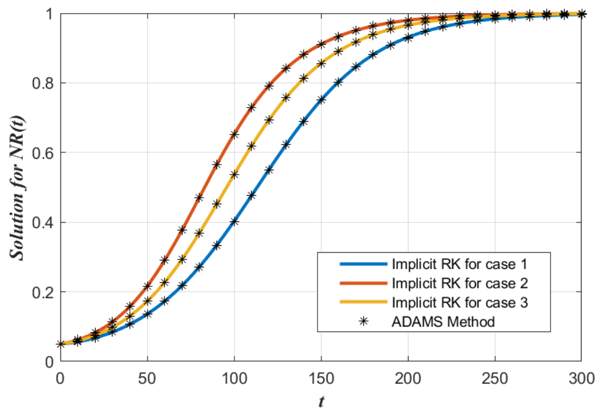

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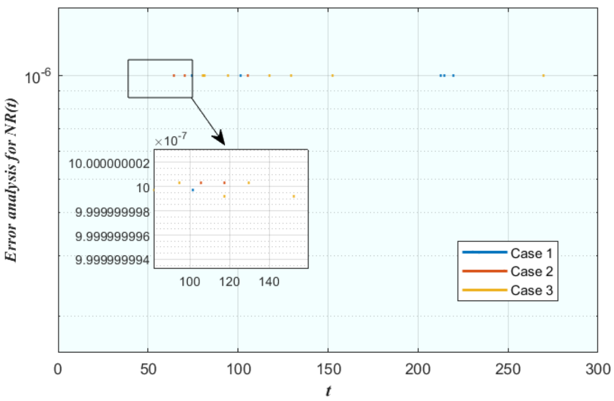

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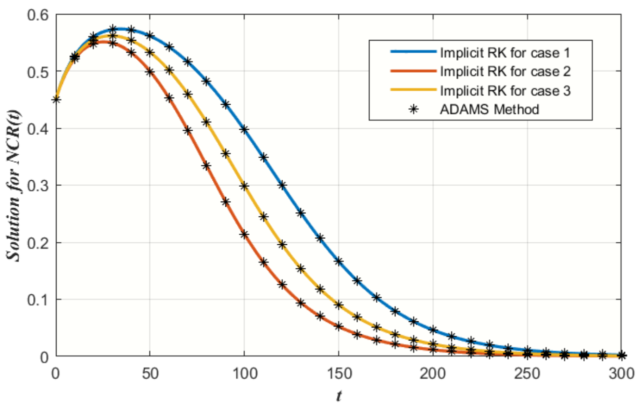

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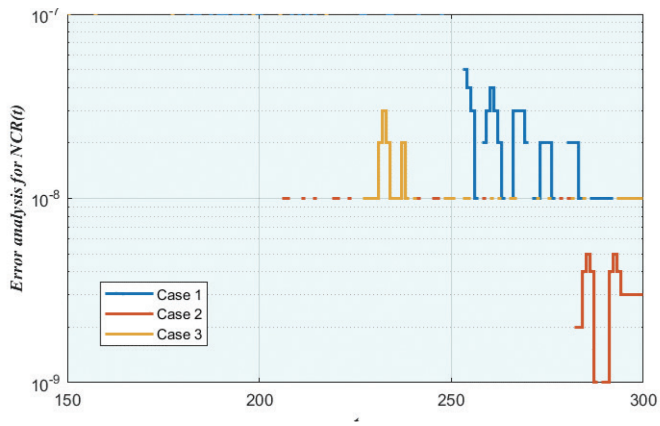

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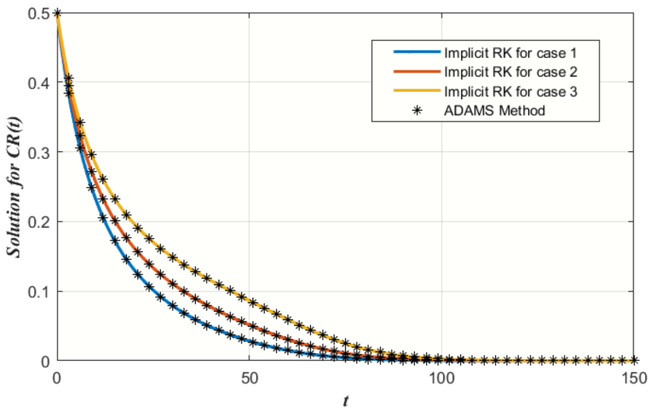
Fig. 6 Solution plots for RAPD model with the employment of Implicit RK numerical technique for case study 3 and associated cases.



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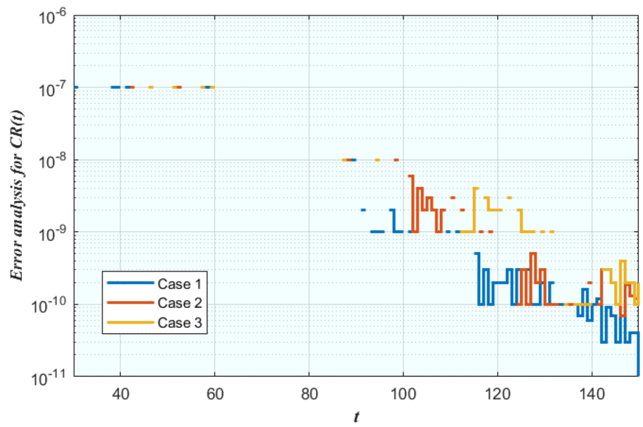

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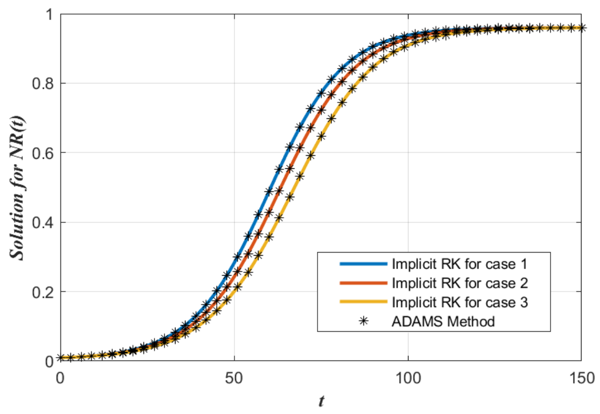

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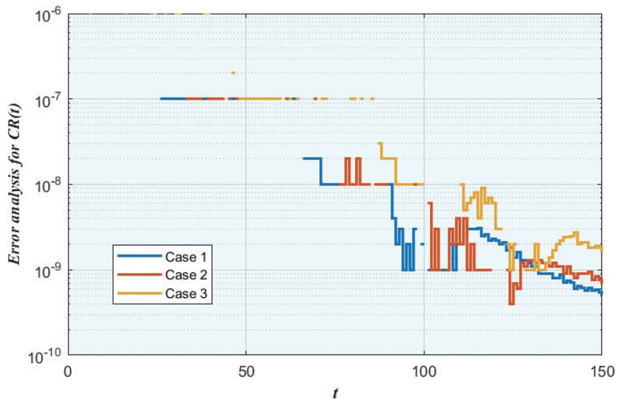

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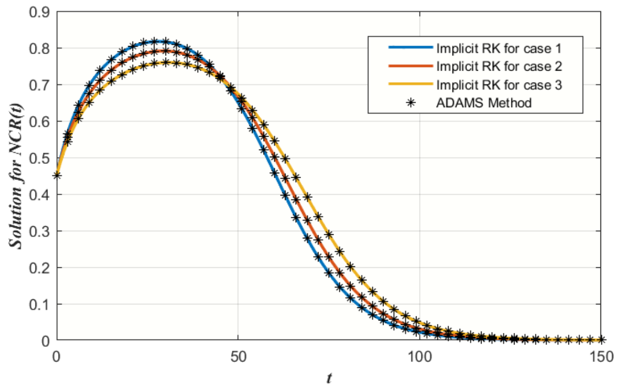

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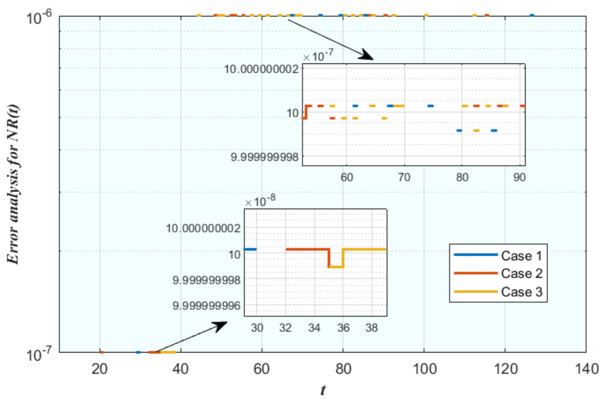

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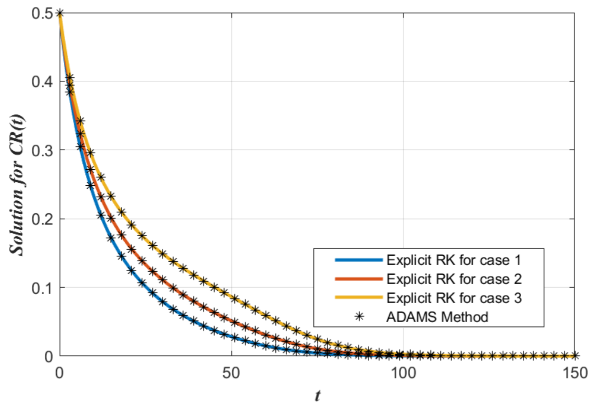
Fig. 7 **Solution plots for RAPD model with the employment of Implicit RK numerical technique for case study 4 and associated cases.**



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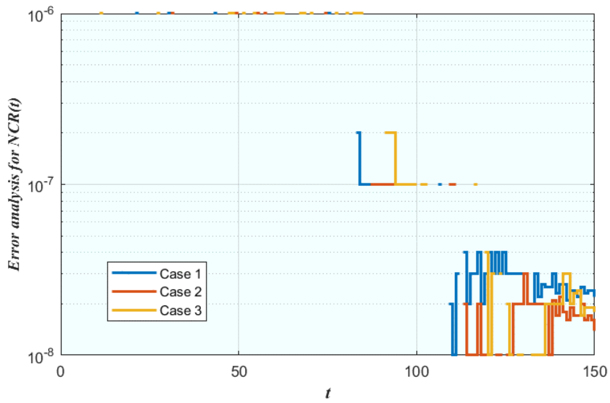

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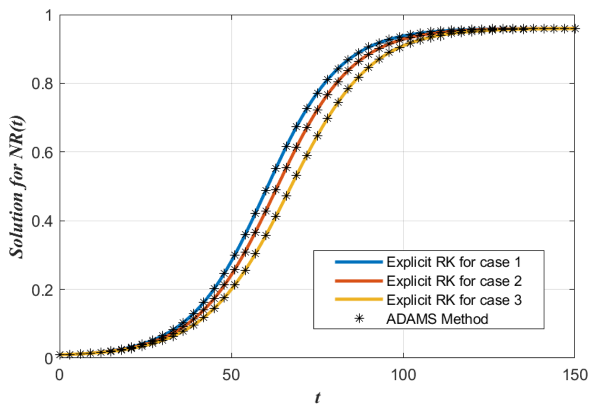

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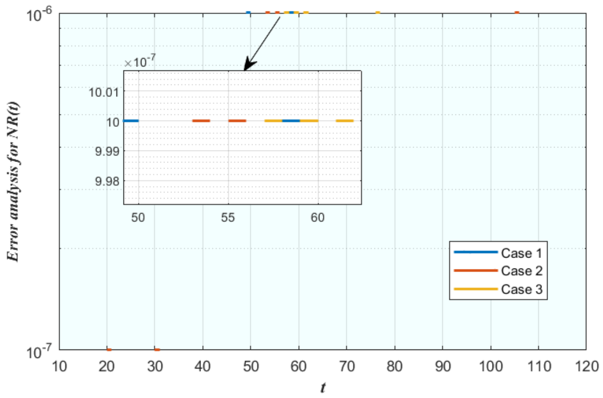

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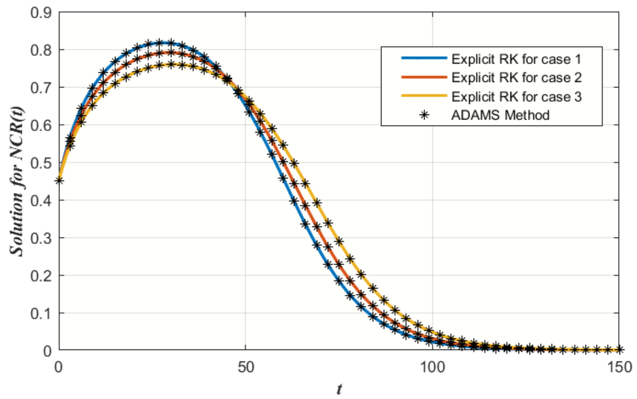

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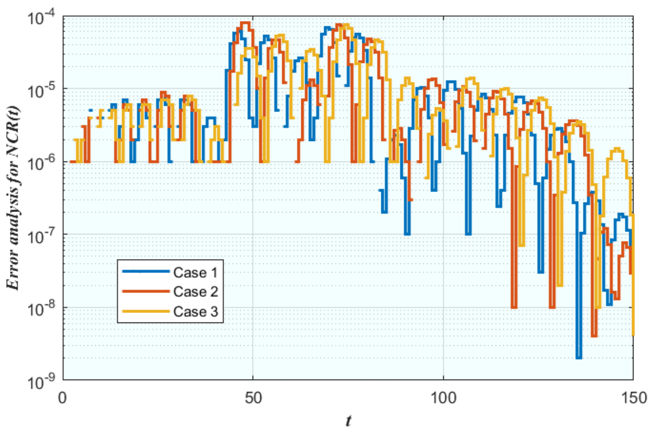
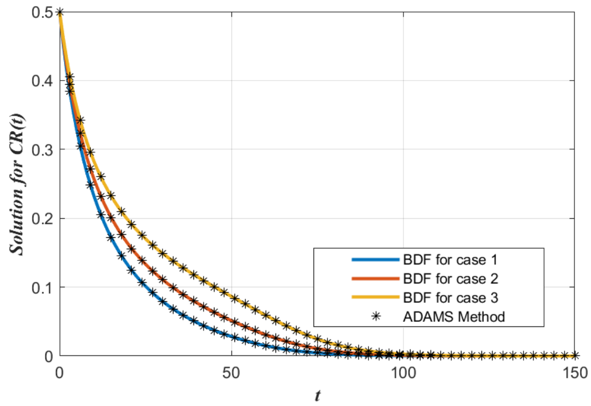


(b:3)

Fig. 8 Solution plots for RAPD model with the employment of Explicit RK numerical technique for case study 4 and associated cases.



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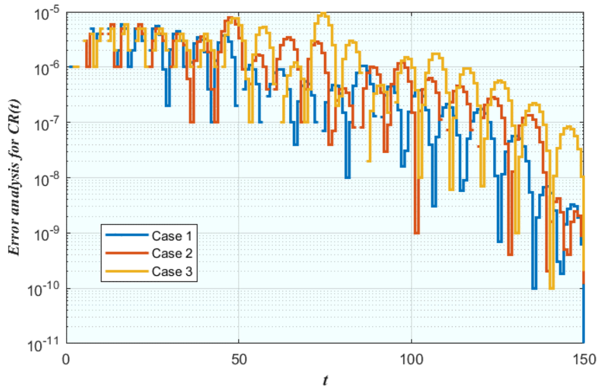

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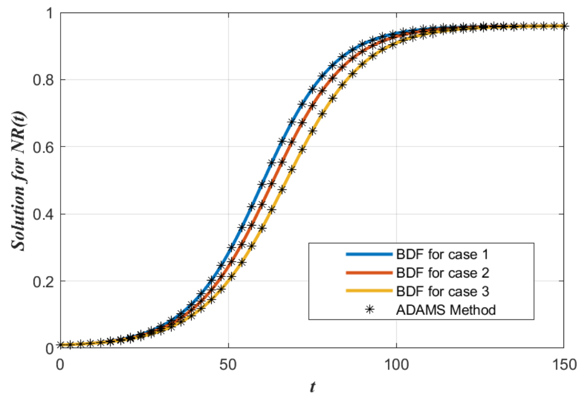

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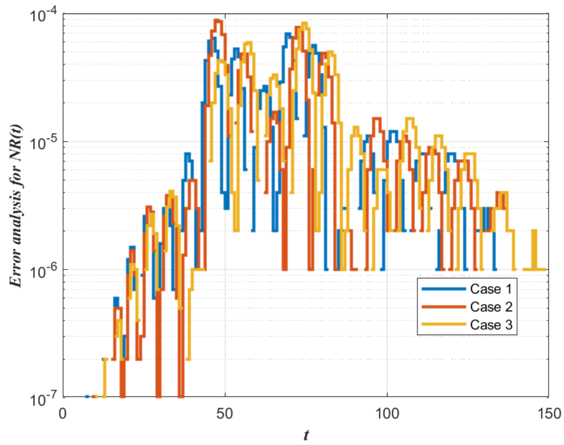

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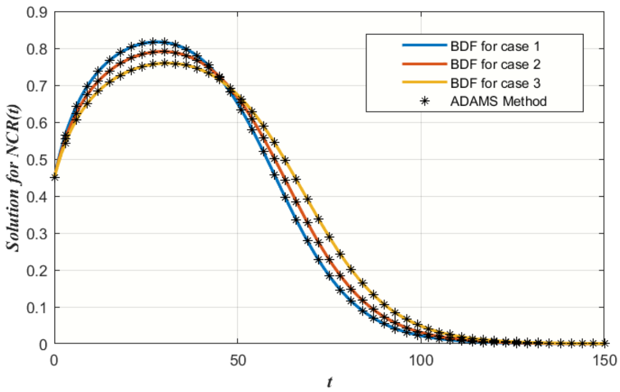

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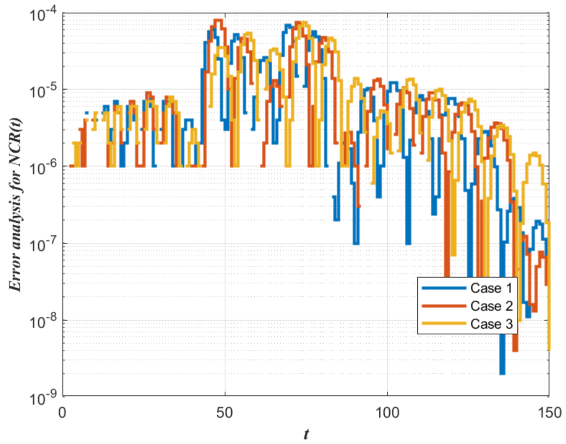

(b:3)

Fig. 9 Solution plots for RAPD model with the employment of BDF numerical technique for case study 4 and associated cases.

4. Conclusion

This presented study highlights the importance of analyzing religious affiliation dynamics in modern social communities through the religious affiliation population dynamics (RAPD) model. By structuring the model with three distinct subpopulations: committed religious (CR), non-committed religious (NCR), and non-religious (NR) individuals along with representing transitions with probability-based social interactions, the RAPD model offers valuable insights into the role's religion plays in human interactions and social cohesion. The application of the Adams predictor-corrector method proved effective in creating a realistic reference dataset that serves as the benchmark for real worl data dynamics for analyzing population shifts. By varying the values of RAPD parameters such as social pressure probabilities and spontaneous transitions, the study examined different case study of religious affiliation changes. Comparative analysis is being carried out using various numerical techniques, including BDF, explicit and implicit Runge-Kutta (RK) methods, that allows for the generation of accurate numerical solutions of the RAPD model. For the evaluation purpose the solution plots along with the error analysis of the state variables i.e., *CR(t)*, *NR(t)*, and *NCR(t)* are illustrated. The results demonstrated that with the exploitation of the different numerical methods mentioned in this study provides reliable accuracy, efficiency and robustness, as indicated by AE values being minimized and consistently remains below 10^{-7} . This suggests that the implemented methods are effective for modeling religious affiliation dynamics. Overall, the RAPD model and the numerical techniques used in this study offer a reliable framework for future investigations into the social implications of religious affiliation and their potential influence on modern communities. The researchers and field experts should employ the novel approaches based on artificial intelligence algorithms [45-51] to obtain accurate, robust and reliable solutions to the complex religious and other social science dynamical models.

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