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# Strategic Energy Planning under Uncertainty: Challenges and Best Practices Supported by a Novel Complex q-Rung Orthopair Fuzzy Rough Decision Framework

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## ABSTRACT

Strategic energy planning faces increasing uncertainty due to fluctuating demand, high renewable integration, technological transitions, and regulatory inconsistencies. These challenges complicate decision-making, often leading to suboptimal investments and policy gaps. This study identifies the key problems in energy strategy formulation, including demand forecasting errors, insufficient flexibility, policy misalignment, and limited stakeholder engagement. To address these issues, a novel complex q-rung orthopair fuzzy rough set (Cq-ROFRS) approach is developed, which is capable of handling uncertainty and aggregating expert information under complex conditions. We have proposed the notion of complex q-rung orthopair fuzzy rough Yager aggregation theory (Yager weighted Average and Geometric). We have delivered an algorithm for a multi-criteria group decision-making approach to utilize the introduced approach. This approach enables systematic assessment and ranking of best practices, integrating both qualitative and quantitative criteria. Results demonstrate that the developed idea provides transparent and adaptable guidance for policymakers and planners. This approach enhances sustainability and effectiveness in energy strategy implementation. Moreover, we have discussed the comparative assessment of the advantages and benefits of the proposed idea.

*Keywords:* Energy Planning and Practices, A Novel Complex q-rung orthopair fuzzy rough set, Decision making

## 1. INTRODUCTION

### 1.1 Energy Planning Background

Strategic energy planning has become complex due to uncertainty in demand forecasts, high penetration of variable renewable energy, and evolving policy targets. These uncertainties collectively challenge conventional deterministic planning paradigms that undermine sustainability and resilience goals [1]. Traditional energy decision-making methods often struggle to handle these uncertainties. The multi-criteria nature of strategic choices leads researchers to adopt advanced fuzzy multi-criteria decision-making (MCDM) frameworks that better capture vagueness and expert hesitancy in judgments [2]. For example, qrung orthopair fuzzy sets (q-ROFS) have been successfully applied to renewable energy source selection and supplier evaluation problems [3]. Systematic reviews also highlight the growing importance of uncertain decision-making models like analytic hierarchy and rough set-based methods, in supporting sustainable energy policy under uncertainty [4]. Alqurashi and Alsager [5] utilized the q-RO neutrosophic fuzzy sets to manage the complex decision-making under uncertainty and proposed the application of these developed approaches. Ezbakhe and Perez-Foguet [6] proposed the Electric III model for the evaluation of renewable energy resources under five criteria.

### 1.2 Literature Review

Zadeh [7] in 1965 initiated the concept of fuzzy set (FS), and it can generalize the crisp set theory. Crisp set theory is all about the belonging and non-belonging of an element to a set. If an element belongs to a set, then its value is regarded as 1 and if it does not, then its membership degree (MD) is 0. There is no other possibility. In FS, the value of . The notion of FS has been regarded as a fundamental tool in many applications and this concept has a fame in the circle of different fields. The application of FS in marine control systems is obvious. To solve the energy decision-making problems, a MCDM approach is proposed and a comprehensive analysis for energy policy making based on fuzzy decision-making methodology has been proposed [8]. Ghomshei et al. [9] proposed the fuzzy methodologies for formulating energy policies. Moreover, the fuzzy values are proposed to evaluate policy parameters. Papapostolou et al. [10] proposed a fuzzy methodology for order preference by similarity to ideal solution for the evaluation of policy scenarios for achieving the 2030 renewable energy scenarios. Kahraman and Kaya [11] presented the fuzzy MCDM methodology for the selection of energy policies. They have proposed the fuzzy AHP technique. Moreover, Soner et al. [12] developed the application of AHP and VIKOR techniques under the interval type 2 fuzzy environment in maritime transportation. Ghommam et al. [13] prescribed a performance-based fuzzy adaptive output

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feedback containment control for multiple under-actuated surface vessels. Many advanced ideas have been produced to show the importance of FS theory and to discuss the MD and NMD in one structure, the notion of the intuitionistic fuzzy set has been developed by Atanassov [14]. In his structure, Atanassov applied a condition that the sum (MD, NMD) must belong to [0, 1]. However, this idea was limited and noted by Yager [15]. Observe that whenever the data was chosen 0.5 as MD and 0.6 as NMD, then sum (0.5, 0.6) does not belong to [0, 1]. So there was a need to develop such an idea that can cover this situation and hence the notion of Pythagorean fuzzy set (PyFS) was produced by Yager [15]. The notion of PyFS uses the advanced constraint that the sum of squares of (MD, NMD) must belong to [0, 1] and we can observe that by using this constraint, the sum of squares of (0.5, 0.6) belongs to [0, 1]. Hence, the idea of PyFS has proven to be a more generalized idea than IFS. Now, Yager also observed that if 0.7 is MD and 0.8 is NMD, then notice that the basic condition of PyFS fails to hold and still there was a need to improve the idea of PyFS. Hence, the notion of q-ROFS has been delivered by Yager [16] and in this case, they have implemented the condition that for Hence, in the case of the above given data, we can observe. Hence, we can regard the notion of q-ROFS as the most advanced idea compared to IFS and PyFS. All of these ideas have been utilized in the maritime system to improve the structure and application of the maritime system. Hao et al. [17] developed context-based distance measures for IFS and used these ideas in marine energy transportation. Also, Mishra et al. [18] developed IF combinative distance measures and used these ideas for sustainability in the maritime sector. Also, Nazligul [19] used the TOPSIS method based on IFS for the selection of a desalination system for commercial vessels. Wang and Sun [20] proposed the mechanism for the implementation of energy engineering projects in China's enterprises. Fu et al. [21] proposed IF entropy and the VIKOR technique with unknown experts and attribute weights. Li et al. [22] used an integrated PyF decision-making approach for risk analysis of offshore wind turbines. Li et al. [23] proposed the TOPSIS technique based on PyFS for the evaluation of dispatch results of the power system. Kamali et al. [24] established the fuzzy SWARA-TOPSIS technique for the evaluation of the EU process toward sustainable energy development. Furthermore, Deveci et al. [25] delivered a hybrid CoCoSo method in the environment of q-ROFS and provided its application in the selection process for a floating offshore wind farm in Norway. The notion of a rough set developed in [26] is a valuable methodical framework that can handle ambiguous and uncertain data effectively. Researchers have combined the notion of RS and FS structure and they have produced different ideas like fuzzy rough set (FRS) [27], IF rough set (IFRS) [28], and PyF rough set (PyFRS) [29] and q-ROF rough set (q-ROFRS) [30]. These ideas have their own importance in FS theory and have different applications. Qahtan et al. [31] developed and analyzed the performance assessment of sustainable transportation in the shipping industry under the notion of q-ROFRS.

## 2. RESEARCH GAPS AND MOTIVATION OF THE PROPOSED WORK

Multiple research gaps in the existing literature need to be covered. The overall discussion is given by

The idea of a complex q-ROFRS was developed by Ali and Mahmood [32] in a polar form where MD and NMD obtain their values from a unit disc in a complex plane rather than a unit square in a complex plane. The chance of data loss increases in the case of existing ideas.

We can notice that all existing literature on energy policies is based on FS, IFS, PyFS and q-ROFS. Although these studies have their own significance and they have been utilized for energy policies and management. But these are limited studies due to their structural restrictions. When the policy makers want to categorize the data into upper and lower approximation, then these existing ideas fail to hold in these situations. Complex and advanced decision-making problems can never be handled by existing studies utilized in the energy sector. There is a need to define such a structure that has the characteristic to discuss the upper and lower approximations of the data.

In the decision-making situation, we need to find such an appropriate structure that can resolve all existing problems and minimize the chance of data loss is less as much as possible. The introduced approach of Tamir's form of complex q-rung orthopair fuzzy rough set (Cq-ROFRS) is very interesting to utilize because it covers the limitation issue of many other ideas. Also, the developed idea can lessen the issues of data loss. In the initiated approach of Cq-ROFRS, the MD and NMD obtain values from the unit square in the complex plane rather than from the unit disc in a complex plane, which makes a significant difference and hence makes the MCDM approach more interesting. It can discuss the MD and NMD in Tamir's form of complex fuzzy numbers. It can cope with the limitations of both intuitionistic fuzzy sets and q-rung orthopair fuzzy sets. Table 1 below shows the needs of the proposed study based on the characteristics of the introduced approach.

**Table 1 Characteristic analysis of the proposed work with existing notions**

| Methodology | Consider the fuzzy framework | Consider the MD and NMD | Consider the Complex fuzzy information | Consideration of upper and lower approximation |
|---|---|---|---|---|
| Wang and Sun [20] approach | Yes | No | No | No |
| Fu et al. [21] approach | Yes | Yes | No | No |
| Li et al. [22] approach | Yes | Yes | No | No |
| Li et al. [23] approach | Yes | Yes | No | No |
| Kamali et al. [24] approach | Yes | Yes | No | No |
| Deveci et al. [25] approach | Yes | Yes | No | No |
| Pawlak [26] approach | No | No | No | Yes |

| Methodology | Consider the fuzzy framework | Consider the MD and NMD | Consider the Complex fuzzy information | Consideration of upper and lower approximation |
|---|---|---|---|---|
| Dubois and Prade [27] approach | Yes | No | No | Yes |
| Zhang et al. [28] approach | Yes | Yes | No | Yes |
| Zhang et al. [29] approach | Yes | Yes | No | Yes |
| Ashraf et al. [30] approach | Yes | Yes | No | Yes |
| Qahtan et al. [31] approach | Yes | Yes | No | Yes |
| Proposed Ideas | Yes | Yes | Yes | Yes |

### 2.1 Main Contribution

This approach delivers a new fuzzy model called Tamir's form of Cq-ROFRS model. Moreover, we have delivered basic operational rules based on Yager's t-norm and t-conorm. We have delivered the idea of Cq-ROFR Yager weighted (average and geometric) Cq-ROFRYWA and Cq-ROFRYWG aggregation operators (AOs). The utilization of the introduced work has been carried out through a MCGDM algorithm. We have used the defined algorithm for the prioritization of practices in the energy sector. We have discovered a comparative study to deliberate the advantages and superiority of the originated work. In the end, we have discussed the concluding remarks.

### 2.2 Section-wise study of the proposed theory

The arrangement of the developed approach is as follows: in Section 1, we have given an overview of the proposed application. Section 2 is about the literature analysis of FS, RS, IFS, PyFS, q-ROFS and application of these ideas to different fields. Moreover, we have discussed the motivation and study framework of the delivered approach. Section 3 is about the preliminary definition that can help us deliver the main notions proposed in this article. In section 4, we have delivered the basic idea of Tamir's form of Cq-ROFRS. Based on the Yager t-norm and t-conorm, we have delivered the Yager operational laws for defining the Yager aggregation theory based on Cq-ROFRSs. Moreover, we have discussed the Cq-ROFRYWA and Cq-ROFRYWG AOs in section 5. In Section 6, we have discussed the algorithm to give evidence about the application of the proposed work. Furthermore, we have discussed the application of the delivered approach for the prioritization of practices in energy systems. In Section 7, we have proposed a comparative analysis of the developed approach that shows the advantages of the introduced approach. In section 8, we have discussed the concluding remarks.

## 3. FUNDAMENTAL NOTIONS

In this section, we will discuss the basic idea of q-ROFS, Yager's operational Laws for q-ROFNS and q-ROFRS.

Definition 1 [16]: Let be the universal set. The notion of q-ROFS is defined by

$$\tilde{\mathrm{N}} = \{\rho, (\theta(\rho), \pi(\rho))|\rho \in Z\}$$

Where $\theta: Z \to [0,1]$ and $\pi: Z \to [0,1]$ are called MD and NMD, respectively, with the condition that $0 \le \theta(\rho)^{\rho} + \pi(\rho)^{\rho} \le 1$ for $\rho \ge 1$.

Definition 2 [39]: Let $\tilde{\mathrm{N}}_1=((\theta_1,\pi_1))$ and $\tilde{\mathrm{N}}_2=((\theta_2,\pi_2))$ be two q-ROFRNs and $r > 0$ and $\tau > 0$. The Yager t-norm and t-conorm operations are as follows

1) $\tilde{\mathrm{N}}_1 \oplus \tilde{\mathrm{N}}_2 = \left\{\left(\sqrt[\rho]{\min\left(1, ((\theta_1)^{\rho r} + (\theta_2)^{\rho r})^{\frac{1}{r}}\right)}, \sqrt[\rho]{1 - \min\left(1, ((1-(\pi_1)^{\rho})^{r} + (1-(\pi_2)^{\rho})^{r})^{\frac{1}{r}}\right)}\right)\right\}$
2) $\tilde{\mathrm{N}}_1 \otimes \tilde{\mathrm{N}}_2 = \left\{\left(\sqrt[\rho]{1 - \min\left(1, ((1-(\theta_1)^{\rho})^{r} + (1-(\theta_2)^{\rho})^{r})^{\frac{1}{r}}\right)}, \sqrt[\rho]{\min\left(1, ((\pi_1)^{\rho r} + (\pi_2)^{\rho r})^{\frac{1}{r}}\right)}\right)\right\}$
3) $\tau\tilde{\mathrm{N}}_1 = \left\{\left(\sqrt[\rho]{\min\left(1, ((\tau\theta_1)^{\rho r})^{\frac{1}{r}}\right)}, \sqrt[\rho]{1 - \min\left(1, (\tau(1-(\pi_1)^{\rho})^{r})^{\frac{1}{r}}\right)}\right)\right\}$
4) $(\tilde{\mathrm{N}}_1)^{\tau} = \left\{\left(\sqrt[\rho]{1 - \min\left(1, (\tau(1-(\theta_1)^{\rho})^{r})^{\frac{1}{r}}\right)}, \sqrt[\rho]{\min\left(1, ((\tau\pi_1)^{\rho r})^{\frac{1}{r}}\right)}\right)\right\}$.

Theorem 1 [39]: Let $\tilde{\mathrm{N}}_1=((\theta_1,\pi_1))$ and $\tilde{\mathrm{N}}_2= \{((\theta_2,\pi_2))\}$ be two q-ROFRNs, then

a) $\tilde{\mathrm{N}}_1 \oplus \tilde{\mathrm{N}}_2 = \tilde{\mathrm{N}}_2 \oplus \tilde{\mathrm{N}}_1$
b) $\tilde{\mathrm{N}}_1 \otimes \tilde{\mathrm{N}}_2 = \tilde{\mathrm{N}}_2 \otimes \tilde{\mathrm{N}}_1$
c) $\tau(\tilde{\mathrm{N}}_1 \oplus \tilde{\mathrm{N}}_2) = \tau\tilde{\mathrm{N}}_1 \oplus \tau\tilde{\mathrm{N}}_2$
d) $(\tau_1 + \tau_2)\tilde{\mathrm{N}}_1 = \tau_1\tilde{\mathrm{N}}_1 \oplus \tau_2\tilde{\mathrm{N}}_1$
e) $(\tilde{\mathrm{N}}_1 \otimes \tilde{\mathrm{N}}_2)^{\tau} = \tilde{\mathrm{N}}_1^{\tau} \otimes \tilde{\mathrm{N}}_2^{\tau}$
f) $\tilde{\mathrm{N}}_1^{\tau_1} \otimes \tilde{\mathrm{N}}_2^{\tau_2} = \tilde{\mathrm{N}}_1^{\tau_1+\tau_2}$

Definition 3 [30]: Assume that $Z$ is a universal set and $\dot{\mathrm{R}}$ a q-ROF relation on $Z$, then pair $(Z,\dot{\mathrm{R}})$. It is said to be a q-ROF approximation space. Assume that $\mathcal{F} = \{(\rho, \mathbb{S}_{\mathcal{F}}(\rho), \mathbb{T}_{\mathcal{F}}(\rho))|\rho \in Z\}$ be a q-ROFS over $Z$. Now, upper (Ur) and lower (Lr) approximations of a set $F$ are given by

$$\overline{\dot{\mathrm{R}}}(\mathcal{F}) = \{(\rho, \lambda(\rho), \mu(\rho))|\rho \in Z\}$$
$$\underline{\dot{\mathrm{R}}}(\mathcal{F}) = \{(\rho, \lambda(\rho), \mu(\rho))|\rho \in Z\}$$

Where,

$$\lambda(\rho) = \mathrm{V}_{\rho \in Z}[\mathfrak{a}(\rho, \mathscr{g}) \mathrm{V} \mathbb{S}_{\mathcal{F}}(\rho)]$$
$$\mu(\rho) = \Lambda_{\rho \in Z}[\mathfrak{c}(\rho, \mathscr{g}) \Lambda \mathbb{T}_{\mathcal{F}}(\rho)]$$

With $0 \le \lambda(\rho)^{\rho} + \mu(\rho)^{\rho} \le 1$.

The pair $\dot{\mathrm{R}}(\mathcal{F}) = \left(\overline{\dot{\mathrm{R}}}(\mathcal{F}), \underline{\dot{\mathrm{R}}}(\mathcal{F})\right) = \left\{\left(\rho, \left(\lambda(\rho), \mu(\rho)\right)\right)\middle|\rho \in Z\right\}$. It is called q-ROFRS.

Definition 4 [40]: Suppose $Z \neq \phi$ . The Tamir's form of complex fuzzy set is defined to be

$$\tilde{\mathrm{N}} = \{\rho, (\theta(\rho))|(\theta(\rho)) = \mathbb{S}_{\tilde{\mathrm{N}}}(\rho) + \iota\mathbb{S}'_{\tilde{\mathrm{N}}}(\rho)\}$$

Where $\mathbb{S}_{\tilde{\mathrm{N}}}(\rho)$ and $\mathbb{S}'_{\tilde{\mathrm{N}}}(\rho) \in [0,1]$. They are called the real and imaginary parts of MD.

## 4. A NOVEL COMPLEX Q-RUNG ORTHO-PAIR FUZZY ROUGH SET (CQ-ROFRS) IN TAMIR'S FORM

Energy systems, being dynamic in nature, require the use of complex numbers, which have both real and imaginary parts that involve complex-valued membership and non-membership, respectively. For example, although an energy storage system could fully match the existing condition (real part), it is also possible that it can be influenced by other factors like seasons and renewable energy (imaginary component). In addition, the non-membership degree could be an indicator of how much the current restrictions and possible risks that may come in the future affect an alternative. The proposed approach provides a more accurate depiction of multidimensional uncertainty and retains decision information, incorporating the idea of complex-valued membership and non-membership. Consequently, the decision maker will have a more comprehensive analysis of energy sources. As such, the decision-making process becomes more realistic and reliable compared to conventional real-valued approaches.

In this part of the article, we have to initiate the notion of Cq-ROFRS. For this purpose, we will discuss the notion of Cq-ROF relation and then, based on this notion, we will develop the idea of Cq-ROFRS. Moreover, to discuss the aggregation theory of the proposed work, we will examine the fundamental operational laws.

The notation used for the universal set and Cq-ROFR relation is given by $Z$ and "$\dot{R}$" respectively.

The overall discussion is given by:

Definition 5: Assume that $Z$ denotes the universal set, then any Cq-ROF subset $\dot{R}$ is called a Cq-ROF relation on given by

$$\dot{R} = \{(\rho, g), \lambda(\rho, g), \mu(\rho, g) | \lambda(\rho, g) = \mathfrak{a}(\rho, g) + \iota\mathbb{b}(\rho, g),\ \mu(\rho, g) = \mathbb{c}(\rho, g) + \iota\mathcal{d}(\rho, g)\},$$

Here $\lambda(\rho,g)$ and $\mu(\rho,g)$ are called MD and NMD, respectively, and $\lambda: Z\times Z\to[0,1]+\iota[0,1]$ and $\mu: Z\times Z\to[0,1]+\iota[0,1]$ satisfies

$$0 \le (\mathfrak{a}(\rho, g))^{\rho} + (\mathbb{c}(\rho, g))^{\rho} \le 1$$

$$0 \le (\mathbb{b}(\rho, g))^{\rho} + (\mathcal{d}(\rho, g))^{\rho} \le 1$$

for $\rho \ge 1$.

Definition 6: Let $Z \neq \emptyset$ and $\dot{R}$ be a Cq-ROF relation on $Z$, then (Z,$\dot{R}$) is called the Cq-ROF approximation space. The set $\mathcal{F} = \{(\rho, \mathbb{S}_{\mathcal{F}}(\rho) + \iota\mathbb{S}'_{\mathcal{F}}(\rho), \mathbb{T}_{\mathcal{F}}(\rho) + \iota\mathbb{T}'_{\mathcal{F}}(\rho)) | \rho \in Z\}$ is a Cq-ROFS over $Z$. Now, Ur and Lr approximations of a set $F$ are given by

$$\overline{\dot{R}}(\mathcal{F}) = \left\{\left(\rho, \lambda_{\overline{\dot{R}}}(\rho), \mu_{\overline{\dot{R}}}(\rho)\right) | \rho \in Z\right\}$$

$$\underline{\dot{R}}(\mathcal{F}) = \left\{\left(\rho, \lambda_{\underline{\dot{R}}}(\rho), \mu_{\underline{\dot{R}}}(\rho)\right) | \rho \in Z\right\}$$

Where,

$$\lambda_{\overline{\dot{R}}}(\rho) = \bigvee_{g\in Z}[\mathfrak{a}(\rho, g) \wedge \mathbb{S}_{\mathcal{F}}(g)] + \iota\bigvee_{g\in Z}[\mathbb{b}(\rho, g) \wedge \mathbb{S}'_{\mathcal{F}}(g)] = \theta_{\overline{\dot{R}}} + \iota\vartheta_{\overline{\dot{R}}}$$

$$\mu_{\overline{\dot{R}}}(\rho) = \bigwedge_{g\in Z}[1 - \mathbb{c}(\rho, g) \vee \mathbb{T}_{\mathcal{F}}(g)] + \iota\bigwedge_{g\in Z}[1 - \mathcal{d}(\rho, g) \vee \mathbb{T}'_{\mathcal{F}}(g)] = \pi_{\overline{\dot{R}}} + \iota\tau_{\overline{\dot{R}}}$$

With

$0 \le (\theta_{\overline{\dot{R}}})^{\rho} + (\pi_{\overline{\dot{R}}})^{\rho} \le 1$ and $0 \le (\vartheta_{\overline{\dot{R}}})^{\rho} + (\tau_{\overline{\dot{R}}})^{\rho} \le 1$ for $\rho \ge 1$.

And

$$\lambda_{\underline{\dot{R}}}(\rho) = \bigwedge_{g\in Z}[1 - \mathfrak{a}(\rho, g) \vee \mathbb{S}_{\mathcal{F}}(g)] + \iota\bigwedge_{g\in Z}[1 - \mathbb{b}(\rho, g) \vee \mathbb{S}'_{\mathcal{F}}(g)] = \theta_{\underline{\dot{R}}} + \iota\vartheta_{\underline{\dot{R}}}$$

$$\mu_{\underline{\dot{R}}}(\rho) = \bigvee_{g\in Z}[\mathbb{c}(\rho, g) \wedge \mathbb{T}_{\mathcal{F}}(g)] + \iota\bigvee_{g\in Z}[\mathcal{d}(\rho, g) \wedge \mathbb{T}'_{\mathcal{F}}(g)] = \pi_{\underline{\dot{R}}} + \iota\tau_{\underline{\dot{R}}}$$

With the conditions of and for $0 \le (\theta_{\overline{\dot{R}}})^{\rho} + (\pi_{\overline{\dot{R}}})^{\rho} \le 1$ and $0 \le (\vartheta_{\overline{\dot{R}}})^{\rho} + (\tau_{\overline{\dot{R}}})^{\rho} \le 1$ for $\rho \ge 1$.

The pair $\dot{R}(\mathcal{F}) = \left(\underline{\dot{R}}(\mathcal{F}), \overline{\dot{R}}(\mathcal{F})\right) = \left\{\left(\rho, \left(\lambda_{\underline{\dot{R}}}(\rho), \mu_{\underline{\dot{R}}}(\rho)\right), \left(\lambda_{\overline{\dot{R}}}(\rho), \mu_{\overline{\dot{R}}}(\rho)\right)\right) \middle| \rho \in Z\right\}$ is alled the Cq-ROFRS with respect (Z,$\dot{R}$). For simplicity, we can say that $\dot{R}(\mathcal{F}) = \left(\underline{\dot{R}}(\mathcal{F}), \overline{\dot{R}}(\mathcal{F})\right) = \left(\left(\theta_{\underline{\dot{R}}} + \iota\vartheta_{\underline{\dot{R}}}, \pi_{\underline{\dot{R}}} + \iota\tau_{\underline{\dot{R}}}\right)\right)$ represent the Cq-ROFRN in Tamir's form.

Definition 7: Let $\tilde{N} = \left(\left(\theta_{\underline{\dot{R}}} + \iota\vartheta_{\underline{\dot{R}}}, \pi_{\underline{\dot{R}}} + \iota\tau_{\underline{\dot{R}}}\right), \left(\theta_{\overline{\dot{R}}} + \iota\vartheta_{\overline{\dot{R}}}, \pi_{\overline{\dot{R}}} + \iota\tau_{\overline{\dot{R}}}\right)\right)$ be a Cq-ROFRN, the score function and accuracy function are

$$Scr.(\tilde{N}) = \frac{1}{8}\left(4 + \left(\theta_{1_{\overline{\dot{R}}}}\right)^{\rho} + \left(\theta_{1_{\underline{\dot{R}}}}\right)^{\rho} + \left(\vartheta_{1_{\overline{\dot{R}}}}\right)^{\rho} + \left(\vartheta_{1_{\underline{\dot{R}}}}\right)^{\rho} - \left(\pi_{1_{\overline{\dot{R}}}}\right)^{\rho} - \left(\pi_{1_{\underline{\dot{R}}}}\right)^{\rho} - \left(\tau_{1_{\overline{\dot{R}}}}\right)^{\rho} - \left(\tau_{1_{\underline{\dot{R}}}}\right)^{\rho}\right) \text{ and } Scr.(\tilde{N}) \in [-1, 1]$$

$$Acr.(\tilde{N}) = \frac{1}{8}\left(4 + \left(\theta_{1_{\overline{\dot{R}}}}\right)^{\rho} + \left(\theta_{1_{\underline{\dot{R}}}}\right)^{\rho} + \left(\vartheta_{1_{\overline{\dot{R}}}}\right)^{\rho} + \left(\vartheta_{1_{\underline{\dot{R}}}}\right)^{\rho} + \left(\pi_{1_{\overline{\dot{R}}}}\right)^{\rho} + \left(\pi_{1_{\underline{\dot{R}}}}\right)^{\rho} + \left(\tau_{1_{\overline{\dot{R}}}}\right)^{\rho} + \left(\tau_{1_{\underline{\dot{R}}}}\right)^{\rho}\right) \text{ and } Acr.(\tilde{N}) \in [0, 1]$$

## 5. TAMIR'S FORM CQ-ROFR AGGREGATION THEORY

Definition 8: Let Let $\tilde{N}_1 = \left(\left(\theta_{1_{\underline{\dot{R}}}} + \iota\vartheta_{1_{\underline{\dot{R}}}}, \pi_{1_{\underline{\dot{R}}}} + \iota\tau_{1_{\underline{\dot{R}}}}\right), \left(\theta_{1_{\overline{\dot{R}}}} + \iota\vartheta_{1_{\overline{\dot{R}}}}, \pi_{1_{\overline{\dot{R}}}} + \iota\tau_{1_{\overline{\dot{R}}}}\right)\right)$ and $\tilde{N}_2 = \left\{\left(\left(\theta_{2_{\underline{\dot{R}}}} + \iota\vartheta_{2_{\underline{\dot{R}}}}, \pi_{2_{\underline{\dot{R}}}} + \iota\tau_{2_{\underline{\dot{R}}}}\right), \left(\theta_{2_{\overline{\dot{R}}}} + \iota\vartheta_{2_{\overline{\dot{R}}}}, \pi_{2_{\overline{\dot{R}}}} + \iota\tau_{2_{\overline{\dot{R}}}}\right)\right)\right\}$ be two Cq-ROFRNs and r>0 and s>0. The operational laws are

1) $\tilde{N}_1 \oplus \tilde{N}_2 = \left\{\begin{pmatrix} \sqrt[\rho]{\min\left(1, \left(\left(\theta_{1_{\underline{\dot{R}}}}\right)^{\rho r} + \left(\theta_{2_{\underline{\dot{R}}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)} + \iota\sqrt[\rho]{\min\left(1, \left(\left(\vartheta_{1_{\underline{\dot{R}}}}\right)^{\rho r} + \left(\vartheta_{2_{\underline{\dot{R}}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1 - \min\left(1, \left(\left(1 - \left(\pi_{1_{\underline{\dot{R}}}}\right)^{\rho}\right)^{r} + \left(1 - \left(\pi_{2_{\underline{\dot{R}}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} + \\ \iota\sqrt[\rho]{1 - \min\left(1, \left(\left(1 - \left(\tau_{1_{\underline{\dot{R}}}}\right)^{\rho}\right)^{r} + \left(1 - \left(\tau_{2_{\underline{\dot{R}}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{\min\left(1, \left(\left(\theta_{1_{\overline{\dot{R}}}}\right)^{\rho r} + \left(\theta_{2_{\overline{\dot{R}}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)} + \iota\sqrt[\rho]{\min\left(1, \left(\left(\vartheta_{1_{\overline{\dot{R}}}}\right)^{\rho r} + \left(\vartheta_{2_{\overline{\dot{R}}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1 - \min\left(1, \left(\left(1 - \left(\pi_{1_{\overline{\dot{R}}}}\right)^{\rho}\right)^{r} + \left(1 - \left(\pi_{2_{\overline{\dot{R}}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} + \\ \iota\sqrt[\rho]{1 - \min\left(1, \left(\left(1 - \left(\tau_{1_{\overline{\dot{R}}}}\right)^{\rho}\right)^{r} + \left(1 - \left(\tau_{2_{\overline{\dot{R}}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}\right\}$

2) $\tilde{N}_1 \otimes \tilde{N}_2 = \left\{ \begin{pmatrix} \sqrt[\rho]{1-\min\left(1,\left(\left(1-\left(\theta_{1_{\underline{R}}}\right)^{\rho}\right)^{r}+\left(1-\left(\theta_{2_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+ \\ \iota\sqrt[\rho]{1-\min\left(1,\left(\left(1-\left(\vartheta_{1_{\underline{R}}}\right)^{\rho}\right)^{r}+\left(1-\left(\vartheta_{2_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{\min\left(1,\left(\left(\pi_{1_{\underline{R}}}\right)^{\rho r}+\left(\pi_{2_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\left(\tau_{1_{\underline{R}}}\right)^{\rho r}+\left(\tau_{2_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{1-\min\left(1,\left(\left(1-\left(\theta_{1_{\overline{R}}}\right)^{\rho}\right)^{r}+\left(1-\left(\theta_{2_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+ \\ \iota\sqrt[\rho]{1-\min\left(1,\left(\left(1-\left(\vartheta_{1_{\overline{R}}}\right)^{\rho}\right)^{r}+\left(1-\left(\vartheta_{2_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{\min\left(1,\left(\left(\pi_{1_{\overline{R}}}\right)^{\rho r}+\left(\pi_{2_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\left(\tau_{1_{\overline{R}}}\right)^{\rho r}+\left(\tau_{2_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\}$

3) $\mathfrak{s}\tilde{N}_1 = \left\{ \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\left(\mathfrak{s}\theta_{1_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\left(\mathfrak{s}\vartheta_{1_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathfrak{s}\left(1-\left(\pi_{1_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+ \\ \iota\sqrt[\rho]{1-\min\left(1,\left(\mathfrak{s}\left(1-\left(\tau_{1_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\left(\mathfrak{s}\theta_{1_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\left(\mathfrak{s}\vartheta_{1_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathfrak{s}\left(1-\left(\pi_{1_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+ \\ \iota\sqrt[\rho]{1-\min\left(1,\left(\mathfrak{s}\left(1-\left(\tau_{1_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\},$

4) $(\tilde{N}_1)^{r} = \left\{ \begin{pmatrix} \sqrt[\rho]{1-\min\left(1,\left(\mathfrak{s}\left(1-\left(\theta_{1_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathfrak{s}\left(1-\left(\vartheta_{1_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{\min\left(1,\left(\left(\mathfrak{s}\pi_{1_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\left(\mathfrak{s}\tau_{1_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{1-\min\left(1,\left(\mathfrak{s}\left(1-\left(\theta_{1_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathfrak{s}\left(1-\left(\vartheta_{1_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{\min\left(1,\left(\left(\mathfrak{s}\pi_{1_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\left(\mathfrak{s}\tau_{1_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\}$

Definition 9: Let $\tilde{N}_i = \left\{\left(\left(\theta_{i_{\underline{R}}}+\iota\vartheta_{i_{\underline{R}}},\pi_{i_{\underline{R}}}+\iota\tau_{i_{\underline{R}}}\right),\left(\theta_{i_{\overline{R}}}+\iota\vartheta_{i_{\overline{R}}},\pi_{i_{\overline{R}}}+\iota\tau_{i_{\overline{R}}}\right)\right)\right\}$ $(i=1,2,\ldots,m)$ be the family of Cq-ROFRNs. The idea of the Cq-ROFRYWA operator is a function $^{\circ}F: \mathbb{N}^{m} \to N$ such that

$$Cq-ROFRYWA_{\mathbb{W}}(\tilde{N}_1,\tilde{N}_2,\tilde{N}_3,\ldots,\tilde{N}_m) = \oplus_{i=1}^{m}(\mathbb{W}_i\tilde{N}_i)$$

Where $\mathbb{W}=(\mathbb{W}_1,\mathbb{W}_2,\mathbb{W}_3,\ldots,\mathbb{W}_m)^{T}$ is the weight vector (WV) of Cq-ROFRNs and $\sum_{i=1}^{m}\mathbb{W}_i = 1$.

Theorem 2: Let $\tilde{N}_i = \left\{\left(\left(\theta_{i_{\underline{R}}}+\iota\vartheta_{i_{\underline{R}}},\pi_{i_{\underline{R}}}+\iota\tau_{i_{\underline{R}}}\right),\left(\theta_{i_{\overline{R}}}+\iota\vartheta_{i_{\overline{R}}},\pi_{i_{\overline{R}}}+\iota\tau_{i_{\overline{R}}}\right)\right)\right\}$ $(i=1,2,\ldots,m)$ be a gathering of Cq-ROFRNs. The evaluated findings by using the Cq-ROFRYWA operator is a Cq-ROFRN defined as $Cq-ROFRYWA_{\mathbb{W}}(\tilde{N}_1,\tilde{N}_2,\tilde{N}_3,\ldots,\tilde{N}_m) = \oplus_{i=1}^{m}(\mathbb{W}_i\tilde{N}_i)$

$$= \left\{ \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\sum_{i=1}^{m}\left(\mathbb{W}_i\left(\theta_{i_{\underline{R}}}\right)^{qr}\right)\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{i=1}^{m}\left(\mathbb{W}_i\left(\vartheta_{i_{\underline{R}}}\right)^{qr}\right)\right)^{\frac{1}{r}}\right)} \\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{i=1}^{m}\left(\mathbb{W}_i\left(1-\left(\pi_{i_{\underline{R}}}\right)^{\rho}\right)^{r}\right)\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{i=1}^{m}\left(\mathbb{W}_i\left(1-\left(\tau_{i_{\underline{R}}}\right)^{\rho}\right)^{r}\right)\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\sum_{i=1}^{m}\left(\mathbb{W}_i\left(\theta_{i_{\overline{R}}}\right)^{\rho r}\right)\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{i=1}^{m}\left(\mathbb{W}_i\left(\vartheta_{i_{\overline{R}}}\right)^{\rho r}\right)\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{i=1}^{m}\left(\mathbb{W}_i\left(1-\left(\pi_{i_{\overline{R}}}\right)^{\rho}\right)^{r}\right)\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{i=1}^{m}\left(\mathbb{W}_i\left(1-\left(\tau_{i_{\overline{R}}}\right)^{\rho}\right)^{r}\right)\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\} \quad (1)$$

Proof: We will prove this structure by mathematical induction

Step 1: For $m=2$,

$$\mathbb{W}_1\tilde{N}_1 = \left\{ \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\theta_{1_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\vartheta_{1_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)} \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\pi_{1_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\tau_{1_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\theta_{1_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\vartheta_{1_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\pi_{1_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\tau_{1_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\} \text{ and}$$

$$\mathbb{W}_2\tilde{N}_2 = \left\{ \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_2\left(\theta_{2_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_2\left(\vartheta_{2_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_2\left(1-\left(\pi_{2_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_2\left(1-\left(\tau_{2_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_2\left(\theta_{2_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_2\left(\vartheta_{2_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_2\left(1-\left(\pi_{2_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_2\left(1-\left(\tau_{2_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\}$$

Therefore, we get

$$\mathbb{W}_1\tilde{N}_1 \oplus \mathbb{W}_2\tilde{N}_2 = \left\{ \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\theta_{1_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\vartheta_{1_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\pi_{1_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\tau_{1_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\theta_{1_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\vartheta_{1_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\pi_{1_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\tau_{1_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\}$$

$$\oplus \left\{ \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_2\left(\theta_{2_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_2\left(\vartheta_{2_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_2\left(1-\left(\pi_{2_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_2\left(1-\left(\tau_{2_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_2\left(\theta_{2_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_2\left(\vartheta_{2_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_2\left(1-\left(\pi_{2_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_2\left(1-\left(\tau_{2_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\}$$

$$= \left\{ \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\theta_{1_{\underline{R}}}\right)^{\rho r}+\mathbb{W}_2\left(\theta_{2_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\vartheta_{1_{\underline{R}}}\right)^{\rho r}+\mathbb{W}_2\left(\vartheta_{2_{\underline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\pi_{1_{\underline{R}}}\right)^{\rho}\right)^{r}+\mathbb{W}_2\left(1-\left(\pi_{2_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+ \\ \iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\tau_{1_{\underline{R}}}\right)^{\rho}\right)^{r}+\mathbb{W}_2\left(1-\left(\tau_{2_{\underline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\theta_{1_{\overline{R}}}\right)^{\rho r}+\mathbb{W}_2\left(\theta_{2_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}+ \\ \iota\sqrt[\rho]{\min\left(1,\left(\mathbb{W}_1\left(\vartheta_{1_{\overline{R}}}\right)^{\rho r}+\mathbb{W}_2\left(\vartheta_{2_{\overline{R}}}\right)^{\rho r}\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\pi_{1_{\overline{R}}}\right)^{\rho}\right)^{r}+\mathbb{W}_2\left(1-\left(\pi_{2_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)}+ \\ \iota\sqrt[\rho]{1-\min\left(1,\left(\mathbb{W}_1\left(1-\left(\tau_{1_{\overline{R}}}\right)^{\rho}\right)^{r}+\mathbb{W}_2\left(1-\left(\tau_{2_{\overline{R}}}\right)^{\rho}\right)^{r}\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\}$$

$$= \left\{ \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\sum_{i=1}^{2}\left(\mathbb{W}_i\left(\theta_{i_{\underline{R}}}\right)^{\rho r}\right)\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{i=1}^{2}\left(\mathbb{W}_i\left(\vartheta_{i_{\underline{R}}}\right)^{\rho r}\right)\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{i=1}^{2}\left(\mathbb{W}_i\left(1-\left(\pi_{i_{\underline{R}}}\right)^{\rho}\right)^{r}\right)\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\sum_{i=1}^{2}\left(\mathbb{W}_i\left(1-\left(\tau_{i_{\underline{R}}}\right)^{\rho}\right)^{r}\right)\right)^{\frac{1}{r}}} \end{pmatrix}, \begin{pmatrix} \sqrt[\rho]{\min\left(1,\left(\sum_{i=1}^{2}\left(\mathbb{W}_i\left(\theta_{i_{\overline{R}}}\right)^{\rho r}\right)\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{i=1}^{2}\left(\mathbb{W}_i\left(\vartheta_{i_{\overline{R}}}\right)^{\rho r}\right)\right)^{\frac{1}{r}}\right)}, \\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{i=1}^{2}\left(\mathbb{W}_i\left(1-\left(\pi_{i_{\overline{R}}}\right)^{\rho}\right)^{r}\right)\right)^{\frac{1}{r}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{i=1}^{2}\left(\mathbb{W}_i\left(1-\left(\tau_{i_{\overline{R}}}\right)^{\rho}\right)^{r}\right)\right)^{\frac{1}{r}}\right)} \end{pmatrix} \right\}$$

Step 2: Suppose equation (1) is valid for m$=k$ that is

$$Cq-ROFRYWA_{\mathrm{W}}(\tilde{\mathrm{N}}_1,\tilde{\mathrm{N}}_2,\tilde{\mathrm{N}}_3,\ldots,\tilde{\mathrm{N}}_{\mathrm{m}})$$

$$=\left\{\begin{pmatrix}\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(\theta_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(\vartheta_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\pi_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\tau_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(\theta_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(\vartheta_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\pi_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\tau_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix}\right\}$$

Now for m$=k+1$, we get

$$Cq-ROFRYWA_{\mathrm{W}}(\tilde{\mathrm{N}}_1,\tilde{\mathrm{N}}_2,\tilde{\mathrm{N}}_3,\ldots,\tilde{\mathrm{N}}_{\mathrm{m}})$$

$$=\left\{\begin{pmatrix}\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(\theta_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(\vartheta_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\pi_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\tau_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(\theta_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(\vartheta_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\pi_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\tau_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix}\right\}$$

$$\oplus\left\{\begin{pmatrix}\sqrt[\rho]{\min\left(1,\left(\left(\mathrm{w}_{k+1}\left(\theta_{k+1\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\left(\mathrm{w}_{k+1}\left(\vartheta_{k+1\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{1-\min\left(1,\left(\left(\mathrm{w}_{k+1}\left(1-\left(\pi_{k+1\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\left(\mathrm{w}_{k+1}\left(1-\left(\tau_{k+1\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}\sqrt[\rho]{\min\left(1,\left(\mathrm{w}_{k+1}\left(\theta_{k+1\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\mathrm{w}_{k+1}\left(\vartheta_{k+1\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{1-\min\left(1,\left(\mathrm{w}_{k+1}\left(1-\left(\pi_{k+1\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\mathrm{w}_{k+1}\left(1-\left(\tau_{k+1\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix}\right\}$$

$$=\left\{\begin{pmatrix}\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k+1}\left(\mathrm{w}_{\mathrm{i}}\left(\theta_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k+1}\left(\mathrm{w}_{\mathrm{i}}\left(\vartheta_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k+1}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\pi_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k+1}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\tau_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k+1}\left(\mathrm{w}_{\mathrm{i}}\left(\theta_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{k+1}\left(\mathrm{w}_{\mathrm{i}}\left(\vartheta_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k+1}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\pi_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{k+1}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\tau_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix}\right\}$$

Equation (1) is valid for m$=k+1$ It is valid for all m.

Definition 10: Let $\tilde{\mathrm{N}}_{\mathrm{i}}=\left\{\left(\left(\theta_{\mathrm{i}\underline{\mathrm{R}}}+\iota\vartheta_{\mathrm{i}\underline{\mathrm{R}}},\pi_{\mathrm{i}\underline{\mathrm{R}}}+\iota\tau_{\mathrm{i}\underline{\mathrm{R}}}\right),\left(\theta_{\mathrm{i}\overline{\mathrm{R}}}+\iota\vartheta_{\mathrm{i}\overline{\mathrm{R}}},\pi_{\mathrm{i}\overline{\mathrm{R}}}+\iota\tau_{\mathrm{i}\overline{\mathrm{R}}}\right)\right)\right\}$ $(\mathrm{i}=1,2,\ldots,\mathrm{m})$ be the family of Cq-ROFRNs. Then the notion of Cq-ROFRYWG AO is

$$Cq-ROFRYWG(\tilde{\mathrm{N}}_1,\tilde{\mathrm{N}}_2,\tilde{\mathrm{N}}_3,\ldots,\tilde{\mathrm{N}}_{\mathrm{m}})=\otimes_{\mathrm{i}=1}^{\mathrm{m}}\left(\tilde{\mathrm{N}}_{\mathrm{i}}^{\mathrm{w}_{\mathrm{i}}}\right)$$

Where $\mathrm{W}=(\mathrm{w}_1,\mathrm{w}_2,\mathrm{w}_3,\ldots,\mathrm{w}_{\mathrm{m}})^{\mathrm{T}}$ is the WVs of $\tilde{\mathrm{N}}_{\mathrm{i}}$ with the condition that $\sum_{\mathrm{i}=1}^{\mathrm{m}}\mathrm{w}_{\mathrm{i}}=1$ and $\mathrm{w}_{\mathrm{i}}>0$.

Theorem 3: Let $\tilde{\mathrm{N}}_{\mathrm{i}}=\left\{\left(\left(\theta_{\mathrm{i}\underline{\mathrm{R}}}+\iota\vartheta_{\mathrm{i}\underline{\mathrm{R}}},\pi_{\mathrm{i}\underline{\mathrm{R}}}+\iota\tau_{\mathrm{i}\underline{\mathrm{R}}}\right),\left(\theta_{\mathrm{i}\overline{\mathrm{R}}}+\iota\vartheta_{\mathrm{i}\overline{\mathrm{R}}},\pi_{\mathrm{i}\overline{\mathrm{R}}}+\iota\tau_{\mathrm{i}\overline{\mathrm{R}}}\right)\right)\right\}$ $(\mathrm{i}=1,2,\ldots,\mathrm{m})$ be the family of Cq-ROFRNs. The result obtained through the Cq-ROFRYWG operator is again Cq-ROFRN.

$$Cq-ROFRYWG_{\mathrm{W}}(\tilde{\mathrm{N}}_1,\tilde{\mathrm{N}}_2,\tilde{\mathrm{N}}_3,\ldots,\tilde{\mathrm{N}}_{\mathrm{m}})=\otimes_{\mathrm{i}=1}^{\mathrm{m}}\left(\tilde{\mathrm{N}}_{\mathrm{i}}^{\mathrm{w}_{\mathrm{i}}}\right)=$$

$$=\left\{\begin{pmatrix}\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\theta_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\theta_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\mathrm{w}_{\mathrm{i}}\left(\pi_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\mathrm{w}_{\mathrm{i}}\left(\tau_{\mathrm{i}\underline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\theta_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{1-\min\left(1,\left(\sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\mathrm{w}_{\mathrm{i}}\left(1-\left(\theta_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho}\right)^{\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)},\\ \sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\mathrm{w}_{\mathrm{i}}\left(\pi_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}+\iota\sqrt[\rho]{\min\left(1,\left(\sum_{\mathrm{i}=1}^{\mathrm{m}}\left(\mathrm{w}_{\mathrm{i}}\left(\tau_{\mathrm{i}\overline{\mathrm{R}}}\right)^{\rho\mathfrak{r}}\right)\right)^{\frac{1}{\mathfrak{r}}}\right)}\end{pmatrix}\right\}$$

Proof: Same as Theorem 2.

Example 1: Let $\tilde{\mathrm{N}}_1=\{((0.5+\iota 0{,}3,0,.2+\iota 0.1),(0.2+\iota 0.7{,}0.4+\iota 0.6))\}$ and $\tilde{\mathrm{N}}_2=\{((0.4+\iota 0.5,0.2+\iota 0.3),(0.3+\iota 0.5{,}0.1+\iota 0.1))\}$. Also assume that $\mathrm{w}_1=0.4$ and $\mathrm{w}_2=0.6$ and and the value of parameter $\mathfrak{r}=2$ and $\rho=3$ Then, the proposed notion of Cq-RPFRYWA AOs is easily implemented as follows

$$=\left(\begin{pmatrix}\sqrt[3]{\min\left(1,((0.4\times(0.5)^{3\times2})+(0.6\times(0.4)^{3\times2}))^{\frac{1}{2}}\right)}+\iota\sqrt[3]{\min\left(1,((0.4\times(0.3)^{3\times2})+(0.6\times(0.5)^{3\times2}))^{\frac{1}{2}}\right)},\\ \sqrt[3]{1-\min\left(1,((0.4\times(1-(0.2)^3)^2)+(0.6\times(1-(0.2)^3)^2))^{\frac{1}{2}}\right)}+\\ \iota\sqrt[3]{1-\min\left(1,((0.4\times(1-(0.1)^3)^2)+(0.6\times(1-(0.3)^3)^2))^{\frac{1}{2}}\right)}\end{pmatrix},\right.$$

$$\left.\begin{pmatrix}\sqrt[3]{\min\left(1,((0.4\times(0.2)^{3\times2})+(0.6\times(0.3)^{3\times2}))^{\frac{1}{2}}\right)}+\iota\sqrt[3]{\min\left(1,((0.4\times(0.7)^{3\times2})+(0.6\times(0.5)^{3\times2}))^{\frac{1}{2}}\right)},\\ \sqrt[3]{1-\min\left(1,((0.4\times(1-(0.4)^3)^2)+(0.6\times(1-(0.1)^3)^2))^{\frac{1}{2}}\right)}+\\ \iota\sqrt[3]{1-\min\left(1,((0.4\times(1-(0.6)^3)^2)+(0.6\times(1-(0.1)^3)^2))^{\frac{1}{2}}\right)}\end{pmatrix}\right)$$

$$=\begin{pmatrix}(0.7491+\iota 0.7454,0.7872+\iota 0.7763),\\ (0.5818+\iota 0.8668,0.7483+\iota 0.7570)\end{pmatrix}$$

## 6. APPLICATION OF THE PROPOSED WORK

### 6.1 Algorithm

Let $\mathrm{H}=\{\mathrm{H}_1,\mathrm{H}_2,\mathrm{H}_3,\ldots,\mathrm{H}_{\mathrm{m}}\}$ be the set of *'m'* alternatives and $\mathrm{H}_{\mathrm{t}}=\{\mathrm{H}_{\mathrm{t}1},\mathrm{H}_{\mathrm{t}2},\mathrm{H}_{\mathrm{t}3},\ldots,\mathrm{H}_{\mathrm{tn}}\}$ represents *'n'* attributes. Assume that there are *"r"* decision-makers $\{\mathcal{H}_1,\mathcal{H}_2,\mathcal{H}_3,\ldots,\mathcal{H}_r\}$ to provide their assessment for $\mathrm{H}_{\mathrm{i}}(\mathrm{i}=1,2,3,\ldots,\mathrm{m})$ and $\mathrm{H}_{tj}(j=1,2,3,..,n)$. Assume $\mathrm{W}=(\mathrm{w}_1,\mathrm{w}_2,\mathrm{w}_3,\ldots,\mathrm{w}_n)^{\mathrm{T}}$ denote the WVs of attributes $\mathrm{H}_{tj}$ and $\mathrm{W}=(\mathrm{w}_1,\mathrm{w}_2,\mathrm{w}_3,\ldots,\mathrm{w}_{\rho})^{\mathrm{T}}$ be the WVs for experts $\mathcal{H}_z(z=1,2,3,\ldots,r)$ such that $\sum_{j=1}^{n}\mathrm{w}_j=1$ and $\sum_{z=1}^{r}\mathrm{w}_z=1$. Based on this initial information, the complete algorithm is

Step 1: Assessment of experts is based on Cq-ROFRNs. The decision matrix is

$$M_{\mathrm{m}\times n}=$$

$$\begin{bmatrix}\left(\begin{pmatrix}\theta_{11\underline{\mathrm{R}}}+\iota\vartheta_{11\underline{\mathrm{R}}},\\ \pi_{11\underline{\mathrm{R}}}+\iota\tau_{11\underline{\mathrm{R}}}\end{pmatrix},\begin{pmatrix}\theta_{11\overline{\mathrm{R}}}+\vartheta_{11\overline{\mathrm{R}}},\\ \pi_{11\overline{\mathrm{R}}}+\iota\tau_{11\overline{\mathrm{R}}}\end{pmatrix}\right) & \left(\begin{pmatrix}\theta_{12\underline{\mathrm{R}}}+\iota\vartheta_{12\underline{\mathrm{R}}},\\ \pi_{12\underline{\mathrm{R}}}+\iota\tau_{12\underline{\mathrm{R}}}\end{pmatrix},\begin{pmatrix}\theta_{12\overline{\mathrm{R}}}+\vartheta_{12\overline{\mathrm{R}}},\\ \pi_{12\overline{\mathrm{R}}}+\iota\tau_{12\overline{\mathrm{R}}}\end{pmatrix}\right) & \cdots & \left(\begin{pmatrix}\theta_{1n\underline{\mathrm{R}}}+\iota\vartheta_{1n\underline{\mathrm{R}}},\\ \pi_{1n\underline{\mathrm{R}}}+\iota\tau_{1n\underline{\mathrm{R}}}\end{pmatrix},\begin{pmatrix}\theta_{1n\overline{\mathrm{R}}}+\vartheta_{1n\overline{\mathrm{R}}},\\ \pi_{1n\overline{\mathrm{R}}}+\iota\tau_{1n\overline{\mathrm{R}}}\end{pmatrix}\right)\\ \left(\begin{pmatrix}\theta_{21\underline{\mathrm{R}}}+\iota\vartheta_{21\underline{\mathrm{R}}},\\ \pi_{21\underline{\mathrm{R}}}+\iota\tau_{21\underline{\mathrm{R}}}\end{pmatrix},\begin{pmatrix}\theta_{21\overline{\mathrm{R}}}+\vartheta_{21\overline{\mathrm{R}}},\\ \pi_{21\overline{\mathrm{R}}}+\iota\tau_{21\overline{\mathrm{R}}}\end{pmatrix}\right) & \left(\begin{pmatrix}\theta_{22\underline{\mathrm{R}}}+\iota\vartheta_{22\underline{\mathrm{R}}},\\ \pi_{22\underline{\mathrm{R}}}+\iota\tau_{22\underline{\mathrm{R}}}\end{pmatrix},\begin{pmatrix}\theta_{22\overline{\mathrm{R}}}+\vartheta_{22\overline{\mathrm{R}}},\\ \pi_{22\overline{\mathrm{R}}}+\iota\tau_{22\overline{\mathrm{R}}}\end{pmatrix}\right) & \cdots & \left(\begin{pmatrix}\theta_{2n\underline{\mathrm{R}}}+\iota\vartheta_{2n\underline{\mathrm{R}}},\\ \pi_{2n\underline{\mathrm{R}}}+\iota\tau_{2n\underline{\mathrm{R}}}\end{pmatrix},\begin{pmatrix}\theta_{2n\overline{\mathrm{R}}}+\vartheta_{2n\overline{\mathrm{R}}},\\ \pi_{2n\overline{\mathrm{R}}}+\iota\tau_{2n\overline{\mathrm{R}}}\end{pmatrix}\right)\\ \vdots & \vdots & & \vdots\\ \left(\begin{pmatrix}\theta_{\mathrm{m}1\underline{\mathrm{R}}}+\iota\vartheta_{\mathrm{m}1\underline{\mathrm{R}}},\\ \pi_{\mathrm{m}1\underline{\mathrm{R}}}+\iota\tau_{\mathrm{m}1\underline{\mathrm{R}}}\end{pmatrix},\begin{pmatrix}\theta_{\mathrm{m}1\overline{\mathrm{R}}}+\vartheta_{\mathrm{m}1\overline{\mathrm{R}}},\\ \pi_{\mathrm{m}1\overline{\mathrm{R}}}+\iota\tau_{\mathrm{m}1\overline{\mathrm{R}}}\end{pmatrix}\right) & \left(\begin{pmatrix}\theta_{\mathrm{m}2\underline{\mathrm{R}}}+\iota\vartheta_{\mathrm{m}2\underline{\mathrm{R}}},\\ \pi_{\mathrm{m}2\underline{\mathrm{R}}}+\iota\tau_{\mathrm{m}2\underline{\mathrm{R}}}\end{pmatrix},\begin{pmatrix}\theta_{\mathrm{m}2\overline{\mathrm{R}}}+\vartheta_{\mathrm{m}2\overline{\mathrm{R}}},\\ \pi_{\mathrm{m}2\overline{\mathrm{R}}}+\iota\tau_{\mathrm{m}2\overline{\mathrm{R}}}\end{pmatrix}\right) & \ddots & \left(\begin{pmatrix}\theta_{\mathrm{mn}\underline{\mathrm{R}}}+\iota\vartheta_{\mathrm{mn}\underline{\mathrm{R}}},\\ \pi_{\mathrm{mn}\underline{\mathrm{R}}}+\iota\tau_{\mathrm{mn}\underline{\mathrm{R}}}\end{pmatrix},\begin{pmatrix}\theta_{\mathrm{mn}\overline{\mathrm{R}}}+\vartheta_{\mathrm{mn}\overline{\mathrm{R}}},\\ \pi_{\mathrm{mn}\overline{\mathrm{R}}}+\iota\tau_{\mathrm{mn}\overline{\mathrm{R}}}\end{pmatrix}\right)\end{bmatrix}$$

Step 2: The Normalization process is carried out by

$$M^{*}_{m\times n} = \begin{cases} \tilde{N}_i \ for\ benefit\ type\ attributes \\ \tilde{N}_i^{c} \ for\ cost\ type\ attributes \end{cases}$$

Where $\tilde{N}_i^{c}$ is the complement of Cq-ROFRNs $\tilde{N}_i$.

Step 3: Find out the combined matrix by using the WV of experts by using the notion of Cq-ROFRYWA or Cq-ROFRYWG AOs. By using these AOs, the overall result will again be Cq-ROFRNs

Step 4: Now, find out the aggregated result by using the notion of Cq-ROFRYWA.

Step 5: Use def. 7 to find out the score values. In case of the same score values for two Cq-ROFRNs, use the accuracy function to arrive at an appropriate result.

Step 6: Rank the alternatives and choose the best result accordingly.

The flowchart of the proposed algorithm is given in Figure 1.

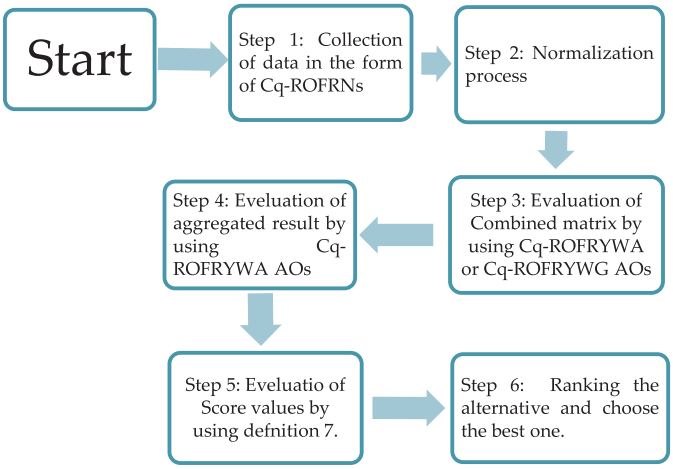



**Fig. 1 The Flowchart of the proposed algorithm**

## 6.2 Illustrative Example

In the energy policy formulation scenario, a government agency or large energy utility must select an optimal energy storage practice to support long-term energy security and high renewable penetration. The decision involves multiple stakeholders' policy makers, grid operators, investors, environmental agencies, and local communities, each expressing preferences using uncertain and linguistic terms such as high scalability, moderate cost stability, acceptable safety risk, inadequacy, and low environmental impact. Future conditions related to renewable intermittency, load growth, technology maturity, regulatory incentives, and climate variability remain highly uncertain, making precise numerical evaluation impractical. Fuzzy theories provide an effective solution by transforming these vague judgments into structured fuzzy representations, allowing degrees of satisfaction, hesitation, and rejection to coexist. Through fuzzy and fuzzy-rough decision frameworks, conflicting stakeholder opinions and incomplete data can be systematically aggregated, enabling robust comparison and prioritization of energy storage options. As a result, decision makers can derive resilient and adaptable energy storage policies that remain effective under evolving uncertainties and long-term strategic constraints.

Traditional decision-making theories rely on precise data, fixed weights, and deterministic assumptions, which are often unrealistic for energy storage selection, where costs, performance, and future demand are uncertain. Such models struggle to represent linguistic evaluations and ignore expert hesitation. They also fail to capture nonlinear interactions among technical, economic, and environmental criteria. Consequently, classical approaches may oversimplify complex trade-offs among storage technologies. This limits their applicability in strategic energy planning under deep uncertainty. To overcome these challenges, we have utilized the proposed algorithm of the MCGDM approach for the selection of the best practice in energy storage systems. There are four alternatives given by

1)Use Advanced Forecasting and Scenario Analysis

The use of forecasts in energy practice allows predicting electricity demand, production of renewable sources of energy, and grid functioning using information from the past and current situation. The analysis of scenarios is aimed at evaluating the performance of energy systems in various alternative situations, including those related to peak demand times, natural disasters, or changes in policies. In general, such analytical methods help handle uncertainties that exist in power generation and transmission and contribute to better system performance.

2)Enhance Grid Stability and Energy Storage Deployment

Some of the strategies that can enhance grid stability through a balance between electricity demand and supply without causing any alterations, power disruption, or frequency challenges include energy storage. Storage of energy involves storing additional energy whenever there is less energy demand, and supplying additional energy whenever there is more demand for energy or if energy is generated intermittently. This is important because it involves smoothing out the surplus energy obtained from sources such as solar and wind energy to guarantee stability in the power system. In this case, efficiency is achieved since power disruptions are avoided, and the integration of renewable energy in the power grid becomes easier. About energy management, forecasting involves using previous records to project future electricity demand, energy supply through renewable sources, and the condition of the power grid. Scenario planning enables one to evaluate other possibilities or futures in terms of the performance of the energy systems.

3)Promote Energy Efficiency as a Strategic Resource

In energy use, energy efficiency is viewed as the "first fuel," whereby efforts to conserve energy usage are as critical as efforts to produce energy. It entails making use of sophisticated technology, efficient processes, and systems to provide the desired energy services with lower energy utilization. In doing so, there will be less reliance on power production, energy transportation, and energy sources, thereby minimizing the impacts on the environment. In essence, energy efficiency is an important strategic energy source.

4)Strengthen the Financial and Risk Management Systems

About practical energy applications, the enhancement of financial and risk management is related to the improvement of the processes of planning, funding, and evaluating energy projects under conditions of risk and uncertainty. In this context, one can mention risks associated with investment that are related to the volatility of fuel prices, changes in policies, demand uncertainties, and challenges in integrating renewables. The use of modern instruments like risk analysis, scenario planning, and probabilistic forecasting makes it possible to reduce potential financial losses.

Based on these alternatives, the attributes are given in Table 2.

**Table 2 Attribute symbols with explanation**

| Symbols | Attributes | Explanation |
|---|---|---|
| $H_{t1}$ | Economic Viability | Economic feasibility is the measure of how economical a particular energy source would be over its entire operational period. It tells us if the chosen energy source is economically viable in terms of whether it will provide satisfactory gains relative to its economic input. |
| $H_{t2}$ | Technical Effectiveness and Reliability | The technical effectiveness of the alternative energy source is based on how effectively the source accomplishes its functions related to generation and efficiency. The reliability of the energy alternative is based on how consistently and steadily the process works over time. |
| $H_{t3}$ | Environmental Sustainability | Sustainability of the environment means the degree to which an energy source avoids any negative effects on the environment through pollution and waste of resources. Sustainability takes into account the ability of an environmental system to achieve balance amid its continued exploitation. |
| $H_{t4}$ | Policy Robustness and Implementation Feasibility | Robustness is the capacity of an energy solution to maintain effectiveness in terms of meeting regulatory, political, and governance changes that occur. The practicality of implementation is the capability of deploying an energy solution despite technological, institutional, and infrastructural challenges. |

Step 1: Suppose professionals evaluate the various alternatives and report the evaluation values in the language terms listed in Table 3-5, where "H" stands for high scalability, "A" acceptable safety risk, "L" low environmental risk, "M" for Moderate cost scalability, and "I" for inadequacy. Moreover, the conversion of linguistic terms into Cq-ROFRNs is provided in Table 3-5 to give meaning to these concepts.

**Table 3 Linguistic data by expert 1**

| | $H_{t1}$ | $H_{t2}$ | $H_{t3}$ | $H_{t4}$ |
|---|---|---|---|---|
| $H_1$ | **H** | **L** | **M** | **H** |
| $H_2$ | **A** | **A** | **I** | **I** |
| $H_3$ | **L** | **H** | **H** | **M** |
| $H_4$ | **M** | **I** | **A** | **L** |

**Table 4 Linguistic data by expert 2**

| | $H_{t1}$ | $H_{t2}$ | $H_{t3}$ | $H_{t4}$ |
|---|---|---|---|---|
| $H_1$ | **L** | **I** | **H** | **L** |
| $H_2$ | **M** | **M** | **A** | **A** |
| $H_3$ | **I** | **L** | **L** | **H** |
| $H_4$ | **H** | **A** | **M** | **I** |

**Table 5 Linguistic data by expert 3**

| | $H_{t1}$ | $H_{t2}$ | $H_{t3}$ | $H_{t4}$ |
|---|---|---|---|---|
| $H_1$ | **I** | **A** | **I** | **I** |
| $H_2$ | **M** | **L** | **M** | **H** |
| $H_3$ | **L** | **I** | **A** | **L** |
| $H_4$ | **A** | **H** | **H** | **M** |

**Table 6 Change of the linguistic information into Cq-ROFRNs**

| Linguistic terms | Cq-ROFNs |
|---|---|
| H | $\left(\begin{pmatrix}0.5+\iota 0.6,\\0.3+\iota 0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota 0.4,\\0.2+\iota 0.1\end{pmatrix}\right)\right)$ |
| L | $\left(\begin{pmatrix}0.1+\iota 0.5,\\0.6+\iota 0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota 0.5,\\0.1+\iota 0.7\end{pmatrix}\right)\right)$ |
| M | $\left(\begin{pmatrix}0.3+\iota 0.4,\\0.7+\iota 0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota 0.4,\\0.3+\iota 0.6\end{pmatrix}\right)\right)$ |
| A | $\left(\begin{pmatrix}0.2+\iota 0.6,\\0.3+\iota 0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota 0.2,\\0.9+\iota 0.1\end{pmatrix}\right)\right)$ |
| I | $\left(\begin{pmatrix}0.1+\iota 0.2,\\0.4+\iota 0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota 0.4,\\0.5+\iota 0.6\end{pmatrix}\right)\right)$ |

We can now go on to the next phase by simply transforming the information provided by the experts in Table 3-5 into Tamir's form of Cq-RORNs. Thus, based on observations of data in Table 6, Table 7-9 exhibit the modified form of the data supplied in Table 3-5.

**Table 7 Changed form of information given in Table 3**

| | $H_{t1}$ | $H_{t2}$ | $H_{t3}$ | $H_{t4}$ |
|---|---|---|---|---|
| $H_1$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.3+\iota0.4,\\0.7+\iota0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.3+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ |
| $H_2$ | $\left(\begin{pmatrix}0.2+\iota0.6,\\0.3+\iota0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.2,\\0.9+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.2+\iota0.6,\\0.3+\iota0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.2,\\0.9+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ |
| $H_3$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.3+\iota0.4,\\0.7+\iota0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.3+\iota0.6\end{pmatrix}\right)\right)$ |
| $H_4$ | $\left(\begin{pmatrix}0.3+\iota0.4,\\0.7+\iota0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.3+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.2+\iota0.6,\\0.3+\iota0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.2,\\0.9+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ |

**Table 8 Changed form of information given in Table 4**

| | $H_{t1}$ | $H_{t2}$ | $H_{t3}$ | $H_{t4}$ |
|---|---|---|---|---|
| $H_1$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ |
| $H_2$ | $\left(\begin{pmatrix}0.3+\iota0.4,\\0.7+\iota0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.3+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.3+\iota0.4,\\0.7+\iota0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.3+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.2+\iota0.6,\\0.3+\iota0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.2,\\0.9+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.2+\iota0.6,\\0.3+\iota0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.2,\\0.9+\iota0.1\end{pmatrix}\right)\right)$ |
| $H_3$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ |
| $H_4$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.2+\iota0.6,\\0.3+\iota0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.2,\\0.9+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.3+\iota0.4,\\0.7+\iota0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.3+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ |

**Table 9 Changed form of information given in Table 5**

| | $H_{t1}$ | $H_{t2}$ | $H_{t3}$ | $H_{t4}$ |
|---|---|---|---|---|
| $H_1$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.2+\iota0.6,\\0.3+\iota0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.2,\\0.9+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ |
| $H_2$ | $\left(\begin{pmatrix}0.3+\iota0.4,\\0.7+\iota0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.3+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.3+\iota0.4,\\0.7+\iota0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.3+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ |

| | $H_{t1}$ | $H_{t2}$ | $H_{t3}$ | $H_{t4}$ |
|---|---|---|---|---|
| $H_3$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.2,\\0.4+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.5+\iota0.6\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.2+\iota0.6,\\0.3+\iota0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.2,\\0.9+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.1+\iota0.5,\\0.6+\iota0.3\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.5,\\0.1+\iota0.7\end{pmatrix}\right)\right)$ |
| $H_4$ | $\left(\begin{pmatrix}0.2+\iota0.6,\\0.3+\iota0.5\end{pmatrix},\left(\begin{pmatrix}0.4+\iota0.2,\\0.9+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.5+\iota0.6,\\0.3+\iota0.2\end{pmatrix},\left(\begin{pmatrix}0.3+\iota0.4,\\0.2+\iota0.1\end{pmatrix}\right)\right)$ | $\left(\begin{pmatrix}0.3+\iota0.4,\\0.7+\iota0.6\end{pmatrix},\left(\begin{pmatrix}0.2+\iota0.4,\\0.3+\iota0.6\end{pmatrix}\right)\right)$ |

Step 2: No need to normalize the decision matrix in Step 1 because of benefit type attributes.

Step 3: We have evaluated the combined matrix through Cq-ROFRYWA AOs with experts WVs $\mathrm{ш} = (0.35, 0{,}36, 0{,}29)^{\mathrm{T}}$ is provided in Table 10.

**Table 10 Combined matrix**

| | $H_{t1}$ | $H_{t2}$ | $H_{t3}$ | $H_{t4}$ |
|---|---|---|---|---|
| $H_1$ | $\left(\begin{pmatrix}0.3183+\iota0.4211,\\0.4196+\iota0.1574\end{pmatrix},\begin{pmatrix}0.1835+\iota0.2900,\\0.1831+\iota0.3703\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.0790+\iota0.4137,\\0.4185+\iota0.3124\end{pmatrix},\begin{pmatrix}0.2056+\iota0.2538,\\0.3437+\iota0.3901\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.3192+\iota0.4197,\\0.5273+\iota0.4184\end{pmatrix},\begin{pmatrix}0.1211+\iota0.2529,\\0.2090+\iota0.3367\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.3183+\iota0.4211,\\0.4196+\iota0.1574\end{pmatrix},\begin{pmatrix}0.1835+\iota0.2900,\\0.1831+\iota0.3703\end{pmatrix}\right)$ |
| $H_2$ | $\left(\begin{pmatrix}0.1574+\iota0.4186,\\0.5609+\iota0.4466\end{pmatrix},\begin{pmatrix}0.1617+\iota0.2070,\\0.3273+\iota0.3401\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.1483+\iota0.4207,\\0.5328+\iota0.4221\end{pmatrix},\begin{pmatrix}0.2056+\iota0.2398,\\0.3101+\iota0.3644\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.1452+\iota0.4196,\\0.5174+\iota0.4138\end{pmatrix},\begin{pmatrix}0.1634+\iota0.2056,\\0.3926+\iota0.3367\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.3123+\iota0.4451,\\0.2283+\iota0.3192\end{pmatrix},\begin{pmatrix}0.1802+\iota0.2056,\\0.3799+\iota0.2362\end{pmatrix}\right)$ |
| $H_3$ | $\left(\begin{pmatrix}0.0316+\iota0.3381,\\0.4453+\iota0.1643\end{pmatrix},\begin{pmatrix}0.2056+\iota0.3179,\\0.1963+\iota0.5352\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.3183+\iota0.4211,\\0.4196+\iota0.1574\end{pmatrix},\begin{pmatrix}0.1835+\iota0.2900,\\0.1831+\iota0.3703\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.3183+\iota0.4460,\\0.4196+\iota0.3124\end{pmatrix},\begin{pmatrix}0.2244+\iota0.2549,\\0.2649+\iota0.2720\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.3192+\iota0.4218,\\0.5314+\iota0.4184\end{pmatrix},\begin{pmatrix}0.1733+\iota0.2829,\\0.1112+\iota0.3610\end{pmatrix}\right)$ |
| $H_4$ | $\left(\begin{pmatrix}0.3192+\iota0.4452,\\0.5272+\iota0.4206\end{pmatrix},\begin{pmatrix}0.1733+\iota0.2153,\\0.2865+\iota0.2362\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.3123+\iota0.4451,\\0.2283+\iota0.3192\end{pmatrix},\begin{pmatrix}0.2294+\iota0.1514,\\0.4743+\iota0.0316\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.3124+\iota0.4445,\\0.5287+\iota0.4221\end{pmatrix},\begin{pmatrix}0.1787+\iota0.2070,\\0.3149+\iota0.2399\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.1451+\iota0.3192,\\0.5233+\iota0.4106\end{pmatrix},\begin{pmatrix}0.1617+\iota0.2890,\\0.2169+\iota0.5012\end{pmatrix}\right)$ |

Step 4: Utilization of Cq-ROFYWA AO to compute the information in Table 10 with attributes WVs (0.21,0.28,0.27,0.24). The evaluated findings are provided in Tables 11 and 12.

**Table 11 Aggregated result by using Cq-ROFRYWA AO**

| $H_1$ | $H_2$ | $H_3$ | $H_4$ |
|---|---|---|---|
| $\left(\begin{pmatrix}\mathbf{0.1740+\iota0.2710},\\\mathbf{0.3377+\iota0.2377}\end{pmatrix},\begin{pmatrix}\mathbf{0.0753+\iota0.1410},\\\mathbf{0.1270+\iota0.2228}\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.1513+\iota0.2799,\\0.3851+\iota0.2735\end{pmatrix},\begin{pmatrix}0.0767+\iota0.1007,\\0.2117+\iota0.1868\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.1756+\iota0.2776,\\0.3398+\iota0.2349\end{pmatrix},\begin{pmatrix}0.0885+\iota0.1530,\\0.0925+\iota0.2419\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.1715+\iota0.2887,\\0.3697+\iota0.2619\end{pmatrix},\begin{pmatrix}0.0837+\iota0.1055,\\0.2065+\iota0.1783\end{pmatrix}\right)$ |

**Table 12 Aggregated result by using Cq-ROFRYWG AO**

| $H_1$ | $H_2$ | $H_3$ | $H_4$ |
|---|---|---|---|
| $\left(\begin{pmatrix}\mathbf{0.1516+\iota0.2708},\\\mathbf{0.2661+\iota0.1686}\end{pmatrix},\begin{pmatrix}\mathbf{0.0842+\iota0.1463},\\\mathbf{0.1774+\iota0.2280}\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.0979+\iota0.2779,\\0.3323+\iota0.2565\end{pmatrix},\begin{pmatrix}0.0833+\iota0.1058,\\0.2265+\iota0.2065\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.1587+\iota0.2651,\\0.3049+\iota0.1621\end{pmatrix},\begin{pmatrix}0.0955+\iota0.1605,\\0.1197+\iota0.3352\end{pmatrix}\right)$ | $\left(\begin{pmatrix}0.1550+\iota0.2710,\\0.3160+\iota0.2468\end{pmatrix},\begin{pmatrix}0.0971+\iota0.1349,\\0.2876+\iota0.3076\end{pmatrix}\right)$ |

Step 5: Use def. 7 of the score function for the data given in Table 11 to find out the score values. The results are given as

$Scr.(\mathrm{H}_1) = 0.5076, Scr.(\mathrm{H}_2) = 0.5106$

$Scr.(\mathrm{H}_3) = 0.5074, Scr.(\mathrm{H}_4) = 0.5101$

Hence, the ranking of the alternative is given by $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. Moreover, Figure 2 shows the graphical representation of score values.

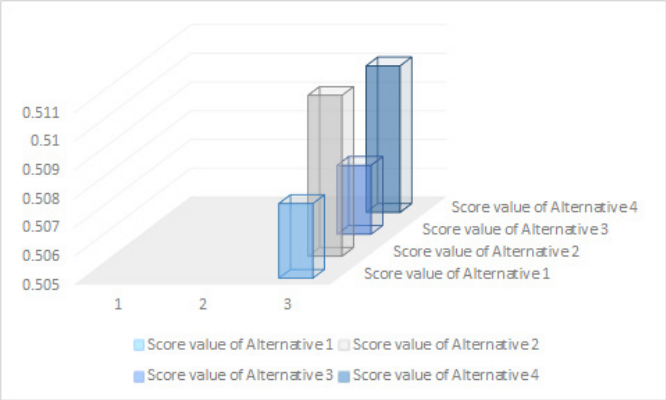



**Fig. 2 Graphical representation of score values**

Also, using the definition. 7 of the score function for the data given in Table 12, we get

$Scr.(\mathrm{H}_1) = 0.5032, Scr.(\mathrm{H}_2) = 0.5067$

$Scr.(\mathrm{H}_3) = 0.5013, Scr.(\mathrm{H}_4) = 0.5017$

Hence, ranking results are given by. Moreover, Figure 3 shows the graphical representation of score values.

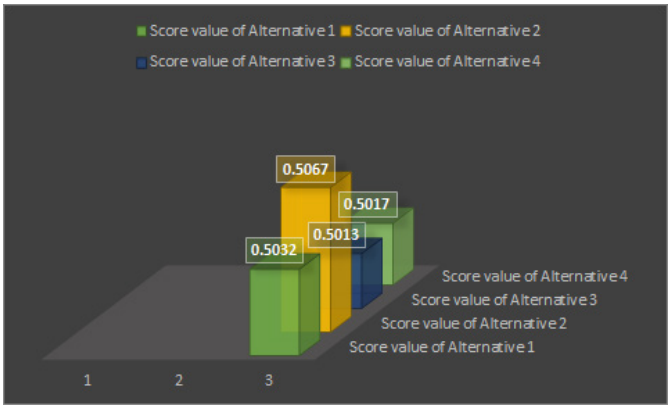



**Fig. 3 Graphical representation of score values**

## 7. COMPARATIVE STUDY

In this part of the article, we will discuss the importance of the developed approach by comparing our initiated work with some other existing notions. The main objective of the comparison is to show how the developed notion would be beneficial and can support advanced information. We will compare our work with Wang and Feng's [33] approach based on Yager AO, Kumar and Chin's [34] idea, based on linguistic IF Yager AO, Kumar and Chin [35] work based on linguistic interval-valued IFS, Mahmood et al. [36] idea of IF rough Yager AOs, Shahzadi et al. [37] approach of PyF Yager AOs, Akram et al. [38] work based on complex PyF Yager AOs and Akram and Shahzadi [39] approach based on q-ROF Yager AOs. In this case, we have used the information from Table 7-9.

The overall discussion is given by

1)The idea proposed by Wang and Feng [33] is based on the simple idea of generalized IF Yager AOs. Note that the IF structure is very limited compared to that of q-ROFS and in this case, we can say that if the information is provided in terms of Cq-ROFRNs, then the settled approach in [33] fails to manage such kind of information, which shows the limitation of the Wang and Feng approach [33]. The introduced work can cover these limitations. Hence introduced work is superior.

2)If we compare our work with Kumar and Chin's approach [34], then we can notice that the chance of information loss increases and the decision-maker's assessment is limited. Decision analysts are bound to take their data in the form of a linguistic IF set and they have developed Yager AOs. We can observe that the limitation of Kumar and Chen's [34] approach is obvious because it cannot cover advanced information in terms of Cq-ROFRNs. The settled approach provides complete independence to decision-makers to take their data freely. So the presented work is superior to the prevailing notion.

3)Although the idea of an interval-valued IF set is more general and the notion defined in [35] is based on interval-valued IF Yager AO, it is a limited notion. These developed philosophies can never cover the two-dimensional evidence in the form of Cq-ROFRNs. Cq-ROFR data consists of upper and lower approximations, making it more advanced in terms of handling two-dimensional information. Cq-ROFR data provide more space for decision analysts to take their information in those applications where the data in terms of Cq-ROFRNs is needed. As developed AOs are based on Cq-ROFRNs, this characteristic makes the originated work more useful than other existing notions.

4)The idea of Yager AOs based on intuitionistic fuzzy rough numbers [36] can cover the issue of upper and lower approximation, but still, it is limited because (1) it cannot cover the two-dimensional information, (2) if a decision analyst proposed his information in terms of Cq-ROFRNs, then the developed approach fails to hold because of its limited structure and condition.

5)Shahzadi et al. [37 ] idea of Pythagorean fuzzy Yager AO is finer than Wang and Feng's [33] approach, but still, it is limited because it can never discuss the rough structure and the chance of data loss increases in this case. This developed structure can never discuss the two-dimensional scenario. The developed approach can discuss all of these limitations. Hence, the initiated work is superior to existing notions.

6)Akram et al. [38] developed an approach based on complex Pythagorean fuzzy Yager AOs, which is based on the polar form of complex fuzzy sets, but the polar form is limited because the MD and NMD belong to the unit disc in the complex plane, rather than the unit square in the complex plane. The introduced approach can cover this kind of issue as well.

7)Akram and Shahzadi [39] approach based on q-ROFNs can cover the issues of IFS and PyFSs, but it can never cover Cq-ROFR information, and this idea can never discuss the two-dimensional information. Hence, the chance of data loss still exists in the theoretical approach. The developed idea can cover these issues.

The overall results based on these observations and using the data from Table 11 and Table 12 are given in Table 13.

**Table 13 Overall result**

| Methods | Score values | Ranking |
|---|---|---|
| Wang and Feng [33] | x | x |
| Kumar and Chen [34] | x | x |
| Kumar and Chen [35] | x | x |
| Mahmood et al. [36] | x | x |
| Shahzadi et al. [37] | x | x |
| Akram et al. [38] | x | x |
| Akram and Shahzadi [39] | x | x |
| Cq-ROFRYWA AOs | $Scr.(\mathrm{H}_1) = 0.5076$, $Scr.(\mathrm{H}_2) = 0.5106$ $Scr.(\mathrm{H}_3) = 0.5074$, $Scr.(\mathrm{H}_4) = 0.5101$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. |
| Cq-ROFRYWG AOs | $Scr.(\mathrm{H}_1) = 0.5032$, $Scr.(\mathrm{H}_2) = 0.5067$ $Scr.(\mathrm{H}_3) = 0.5013$, $Scr.(\mathrm{H}_4) = 0.5017$ | $\mathrm{H}_2 > \mathrm{H}_1 > \mathrm{H}_4 > \mathrm{H}_3$. |

The graphical representation of the comparative analysis is given in Figure 4.

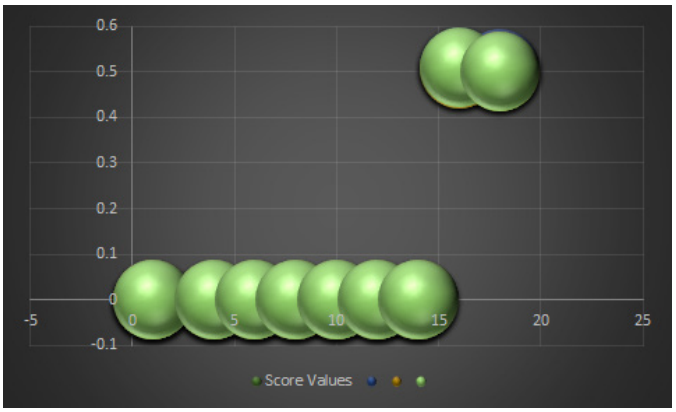



**Fig. 4 Graphical representation of comparative analysis**

### 7.1 Sensitivity Analysis

This section of the article is devoted to discussing the stability of the proposed work. Here, we have used different values of parameters used in Yager aggregation operators. We have analyzed the result and discussed the effect of the parameter on the score values and ranking results. The overall results are given in Tables 14 and 15.

**Table 14 Sensitivity Analysis using Cq-ROFRYWA AOs**

| Parameter Values | Score Values | Ranking Results |
|---|---|---|
| $\mathfrak{r} = 1$ | $Scr.(\mathrm{H}_1) = 0.5467$, $Scr.(\mathrm{H}_2) = 0.5515$ $Scr.(\mathrm{H}_3) = 0.5501$, $Scr.(\mathrm{H}_4) = 0.5746$ | $\mathrm{H}_4 > \mathrm{H}_2 > \mathrm{H}_3 > \mathrm{H}_1$. |
| $\mathfrak{r} = 2$ | $Scr.(\mathrm{H}_1) = 0.5055$, $Scr.(\mathrm{H}_2) = 0.5092$ $Scr.(\mathrm{H}_3) = 0.5054$, $Scr.(\mathrm{H}_4) = 0.5085$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. |
| $\mathfrak{r} = 3$ | $Scr.(\mathrm{H}_1) = 0.5064$, $Scr.(\mathrm{H}_2) = 0.5099$ $Scr.(\mathrm{H}_3) = 0.5062$, $Scr.(\mathrm{H}_4) = 0.5093$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. |
| $\mathfrak{r} = 4$ | $Scr.(\mathrm{H}_1) = 0.50046$, $Scr.(\mathrm{H}_2) = 0.5009$ $Scr.(\mathrm{H}_3) = 0.50044$, $Scr.(\mathrm{H}_4) = 0.5007$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. |
| $\mathfrak{r} = 5$ | $Scr.(\mathrm{H}_1) = 0.5076$, $Scr.(\mathrm{H}_2) = 0.5106$ $Scr.(\mathrm{H}_3) = 0.5074$, $Scr.(\mathrm{H}_4) = 0.5101$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. |

**Table 15 Sensitivity Analysis using Cq-ROFRYWG AOs**

| Parameter Values | Score Values | Ranking Results |
|---|---|---|
| $\mathfrak{r} = 1$ | $Scr.(\mathrm{H}_1) = 0.5219$, $Scr.(\mathrm{H}_2) = 0.5380$ $Scr.(\mathrm{H}_3) = 0.5287$, $Scr.(\mathrm{H}_4) = 0.5416$ | $\mathrm{H}_4 > \mathrm{H}_2 > \mathrm{H}_3 > \mathrm{H}_1$. |
| $\mathfrak{r} = 2$ | $Scr.(\mathrm{H}_1) = 0.5202$, $Scr.(\mathrm{H}_2) = 0.5342$ $Scr.(\mathrm{H}_3) = 0.5213$, $Scr.(\mathrm{H}_4) = 0.5248$ | $\mathrm{H}_4 > \mathrm{H}_2 > \mathrm{H}_3 > \mathrm{H}_1$. |
| $\mathfrak{r} = 3$ | $Scr.(\mathrm{H}_1) = 0.5036$, $Scr.(\mathrm{H}_2) = 0.5073$ $Scr.(\mathrm{H}_3) = 0.5026$ $Scr.(\mathrm{H}_4) = 0.5037$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. |
| $\mathfrak{r} = 4$ | $Scr.(\mathrm{H}_1) = 0.50023$, $Scr.(\mathrm{H}_2) = 0.50064$ $Scr.(\mathrm{H}_3) = 0.50006$, $Scr\ (\mathrm{H}_4) = 0\,\dot{5}0025$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. |
| $\mathfrak{r} = 5$ | $Scr.(\mathrm{H}_1) = 0.50000$, $Scr.(\mathrm{H}_2) = 0.50003$ $Scr.(\mathrm{H}_3) = 0.49999$, $Scr.(\mathrm{H}_4) = 0.50001$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. |

We can observe from Table 14 for Cq-RPFRYWA AOs that for the parameter value $\mathfrak{r} = 1$ the the ranking result is $\mathrm{H}_4 > \mathrm{H}_2 > \mathrm{H}_3 > \mathrm{H}_1$ and for all other values of the parameter $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$. These results show that parameter values are affecting the ranking result and Cq-ROFRYWA AOs are not unstable.

Moreover, when we observe from Table 15, the ranking result for parameter value $\mathfrak{r} = 1, 2$ is $\mathrm{H}_4 > \mathrm{H}_2 > \mathrm{H}_3 > \mathrm{H}_1$ which is stable for these two parameter values. Then the ranking result becomes stale for all other parameter values and their ranking results are $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$.

Similarly, we can do the analysis based on parameter We use the different values of and . We will evaluate the results and check the stability of the Cq-ROFRYWA and Cq-ROFRYWG AOs. The overall results are given in Tables 16 and 17.

**Table 16 Ranking results for different values of by using Cq-ROFRYWA AOs**

| Parameter Values | Score values | Ranking Result for Cq-ROFRYWA AOs |
|---|---|---|
| $\rho = 2$ | $Scr.(\mathrm{H}_1) = 0.5365,$ $Scr.(\mathrm{H}_2) = 0.5458$ $Scr.(\mathrm{H}_3) = 0.5371$ $Scr.(\mathrm{H}_4) = 0.5510$ | $\mathrm{H}_4 > \mathrm{H}_2 > \mathrm{H}_3 > \mathrm{H}_1$ |
| $\rho = 3$ | $Scr.(\mathrm{H}_1) = 0.5064,$ $Scr.(\mathrm{H}_2) = 0.5099$ $Scr.(\mathrm{H}_3) = 0.5062$ $Scr.(\mathrm{H}_4) = 0.5093$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$ |
| $\rho = 4$ | $Scr.(\mathrm{H}_1) = 0.50046,$ $Scr.(\mathrm{H}_2) = 0.50094$ $Scr.(\mathrm{H}_3) = 0.50044$ $Scr.(\mathrm{H}_4) = 0.50078$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_3 > \mathrm{H}_1$ |

**Table 17 Ranking results for different values of by using Cq-ROFRYWG AOs**

| Parameter Values | Score values | Ranking Result for Cq-ROFRYWA AOs |
|---|---|---|
| $\rho = 2$ | $Scr.(\mathrm{H}_1) = 0.5202,$ $Scr.(\mathrm{H}_2) = 0.5342$ $Scr.(\mathrm{H}_3) = 0.5213$ $Scr.(\mathrm{H}_4) = 0.5248$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_3 > \mathrm{H}_1$ |
| $\rho = 3$ | $Scr.(\mathrm{H}_1) = 0.5036,$ $Scr.(\mathrm{H}_2) = 0.5073$ $Scr.(\mathrm{H}_3) = 0.5026$ $Scr.(\mathrm{H}_4) = 0.5037$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$ |
| $\rho = 4$ | $Scr.(\mathrm{H}_1) = 0.5002,$ $Scr.(\mathrm{H}_2) = 0.5006$ $Scr.(\mathrm{H}_3) = 0.5000$ $Scr.(\mathrm{H}_4) = 0.5003$ | $\mathrm{H}_2 > \mathrm{H}_4 > \mathrm{H}_1 > \mathrm{H}_3$ |

Based on the results of Tables 16 and 17, we can notice that in the case of Cq-ROFRYWA and Cq-ROFRYWG AOs, the ranking results are the same for the variation of the parameter. This observation shows the stability of the proposed work.

## 8. CONCLUSION

The utilization of advanced structure is always the preference of decision-makers and it is beneficial to use such a construction where the chance of statistical loss is negligible. The idea of complex IFS in Tamir's form and complex PyFS are limited notions because they cannot ever deliberate the upper and lower approximations. To see the limitations of these fuzzy structures, we have developed the notion of Cq-ROFRS. This idea is free from all those complexities faced by existing notions because a defined notion can discuss the two-dimensional data, as well as can also cover the upper and lower approximations. Furthermore, Yager's functioning rules based on Cq-ROFRNs have been initiated. The idea of Cq-ROFYWA and Cq-ROFRYWG AOs has been introduced. We have introduced an MCGDM approach to discover advanced data. An illustrative example is initiated for the prioritization of energy strategy practices. The comparative analysis determines the advancement and usefulness of the delivered approach.

To discuss the limitations of the developed approach, if the decision-makers want to use an abstinence grade, then the delivered work can never discuss the abstinence grade.

For convenience, the following Table 18 shows the symbols and their terminologies.

**Table 18 Symbols and their terminologies used throughout the article.**

| Symbols | Terminologies |
|---|---|
| $\boldsymbol{r}$ | Yager Parameter |
| $\mathrm{W}$ | Weight vector |
| $\mathrm{H}$ | Alternative |
| $\mathrm{H}_t$ | Attribute |
| $\boldsymbol{\theta}_{\underline{\mathbb{R}}}$ | Lower real part of membership grade |
| $\boldsymbol{\vartheta}_{\underline{\mathbb{R}}}$ | Lower imaginary part of membership grade |
| $\boldsymbol{\pi}_{\underline{\mathbb{R}}}$ | Lower real part of non-membership grade |
| $\boldsymbol{\tau}_{\underline{\mathbb{R}}}$ | Lower imaginary part of non-membership grade |
| $\boldsymbol{\theta}_{\overline{\mathbb{R}}}$ | Upper real part of membership grade |
| $\boldsymbol{\vartheta}_{\overline{\mathbb{R}}}$ | Upper imaginary part of membership grade |
| $\boldsymbol{\pi}_{\overline{\mathbb{R}}}$ | Upper real part of non-membership grade |
| $\boldsymbol{\tau}_{\overline{\mathbb{R}}}$ | Upper imaginary part of non-membership grade |

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